From Perpetual Contracts to Islamic Credit: The Random Stopping Time Equivalence and a Riba-Free Foundation for Sukuk, Takaful, and Credit Protection

Shehzad Ahmed[1] and Rafiqul Bhuyan*[2]

[1]Department of Finance, Independent University, Bangladesh. 2120922@iub.edu.bd
[2]Department of Accounting and Finance, Alabama A&M University, USA. rafiqul.bhuyan@aamu.edu

February 2026 (revised June 2026)

Abstract

We establish a formal mathematical equivalence between two previously unconnected pricing frameworks — the random time representation of perpetual futures contracts [3] and the reduced-form credit risk model of Duffie & Singleton [19] — and demonstrate that this equivalence has far-reaching implications for Islamic finance.

The core observation is structural: the stopping time θ_t in the Ackerer-Hugonnier-Jermann perpetual pricing formula is isomorphic to the default time τ in the Duffie-Singleton defaultable bond formula, under the identification of the convergence intensity κ with the hazard rate λ and the interest parameter ι with the risk-free discount rate r. When the interest parameter is set to zero ($\iota = 0$) — which Ackerer et al. prove achieves unique no-arbitrage perpetual futures pricing — the analogous credit instrument at $r = 0$ is also no-arbitrage, with the stopping time distribution alone carrying all timing information previously encoded in the interest discount.

This equivalence supplies what Islamic finance has lacked for half a century: a mathematically rigorous, no-arbitrage pricing foundation for credit instruments — sukuk, takaful, and credit default swaps — that requires no positive interest rate. We prove the main theorem, derive closed-form prices for an Islamic perpetual sukuk and an everlasting takaful contract using Proposition 6 of Ackerer et al., and show that the convergence intensity κ functions as a riba-free analog to the risk-free rate across all Islamic credit instruments. We term this the **κ-rate**. We further establish a **Separation Theorem**: at $\iota = 0$ the no-arbitrage price carries no interest term, so interest is an *eliminable* component of pricing — not an intrinsic necessity. Empirical evidence from the European negative interest rate environment (2014–2022) corroborates the theoretical finding that credit markets function without a positive discount rate. We further validate the central identification $\hat{\kappa} = s/(1-\delta)$ on real markets: across 195 actively-quoted USD sukuk (40 sovereign, 154 corporate, one municipal; Cbonds), the implied $\hat{\kappa}$ has the predicted credit-regime magnitude (median ≈ 0.02 per annum) and broadly increases as credit quality deteriorates (apart from the Saudi-dominated AA bucket, which prices just above single-A) — so the riba-free intensity is a measurable property of the traded sukuk market, not only a theoretical construct.

Keywords: *riba*, Islamic finance, credit risk, sukuk, takaful, perpetual contracts, stopping time, no-arbitrage, Duffie-Singleton, κ-rate, riba-free pricing, blockchain sukuk

JEL Codes: G12, G13, G15, G22, Z12

1 Introduction

Pricing credit risk is arguably the central unresolved problem in Islamic finance. The \$3 trillion Islamic finance industry [32], which has grown from a niche experiment into a global system over the past half century [30], operates without any mathematical tool for pricing credit default risk that does not require a positive interest rate. Indeed, every credit-derivatives framework in use today — the structural models of Merton [46], Black & Cox [12], and Leland [44]; the reduced-form models of Jarrow & Turnbull [33], Duffie & Singleton [19, 20], and Lando [43]; the textbook synthesis of Bielecki & Rutkowski [9] and Jeanblanc, Yor & Chesney [35] — is built on a positive risk-free interest rate $r > 0$. Islamic scholars correctly identify this r as *riba*: it is the compensation for the pure time value of money, which Islamic jurisprudence prohibits as a predetermined excess in exchange transactions [22, 55].

The consequence has been severe. Islamic institutions cannot hedge credit risk [42]. Sukuk (Islamic bonds) are priced at issuance using LIBOR/SOFR-anchored models, making the interest rate implicit even when explicit interest payments are restructured away [54, 37, 49]. Takaful (Islamic insurance) operators invest pools in sukuk priced by riba-based models [11]. The Islamic credit default swap does not exist. Scholars have sought workarounds for decades, but these have been cosmetic, not mathematical [40, 22].

This paper resolves the impasse. We demonstrate that a recent result from mathematical finance — the random time representation theorem for perpetual futures contracts proved by Ackerer, Hugonnier & Jermann (2025) [3] — contains, as a special case, a complete and rigorous pricing framework for credit instruments that does not require $r > 0$.

The Core Observation

The Ackerer-Hugonnier-Jermann theorem establishes that the unique no-arbitrage price of a perpetual futures contract on a spot asset X_t with interest parameter ι and convergence intensity κ is:

$$f_t = \mathbb{E}_t^{Q^a}\left[\mathrm{e}^{-\int_t^{\theta_t} \iota_s \, ds} X_{\theta_t}\right] \tag{1}$$

where θ_t is a random stopping time with exponential density under the pricing measure Q^a. The critical result, proved in their Theorem 3, is that $\iota = 0$ satisfies all no-arbitrage conditions. When $\iota = 0$, equation (1) collapses to:

$$f_t = \mathbb{E}_t^{Q^a}[X_{\theta_t}], \qquad \theta_t \sim t + \mathrm{Exp}(\kappa) \tag{2}$$

The instrument price is the *expected spot value at a random future moment* — with the probability distribution of that moment determined entirely by κ, and no interest discount anywhere.

The Duffie-Singleton reduced-form credit model prices a defaultable instrument as:

$$P_t = \mathbb{E}_t^{Q}\left[\mathrm{e}^{-\int_t^{\tau} r_s \, ds} R_\tau\right] \tag{3}$$

where τ is the default time (exponentially distributed with hazard rate λ) and R_τ is the recovery value at default.

Equations (1) and (3) are the same equation. The stopping time θ_t is τ. The convergence intensity κ is λ. The interest parameter ι is r. The spot X_{θ_t} is the recovery R_τ.

At $\iota = r = 0$: both yield $\mathbb{E}[X_\tau]$. The Ackerer theorem proves this is no-arbitrage. Therefore credit pricing at $r = 0$ is no-arbitrage, provided $\kappa = \lambda > 0$ carries all timing information. *Riba* is therefore not a pricing necessity but a historical artifact of fusing timing and discounting into a single parameter.

Related Literature

This paper sits at the intersection of four bodies of work.

Reduced-form credit risk. Jarrow & Turnbull [33] introduced intensity-based default modeling; Duffie & Singleton [19] generalized the framework to affine stochastic intensities and established the risk-adjusted short-rate representation; their book [20] provides the comprehensive treatment. Lando [43] gave the definitive Cox-process (doubly stochastic Poisson) treatment. Brémaud [14] established the martingale theory for point processes underpinning survival-probability calculations. Elliott, Jeanblanc & Yor [23] resolved the filtration enlargement problem; Jeanblanc, Yor & Chesney [35] provide the definitive mathematical synthesis. Bielecki & Rutkowski [9] and Schönbucher [50] synthesized this literature into standard practitioner frameworks. Duffie & Lando [18] extended the framework to incomplete accounting information. Our contribution shows that the entire reduced-form machinery transfers to Islamic finance under $\iota = r = 0$, with κ replacing λ.

Perpetual contract theory. Ackerer, Hugonnier & Jermann [3] prove that perpetual futures have unique no-arbitrage prices at $\iota = 0$ and admit the random time representation (Theorem 3) central to our argument. Merton [46] and Leland [44] priced perpetual-maturity corporate debt using structural first-passage models; Black & Cox [12] introduced the stopping-time formalism in that context. Shiller [52] proposed macro perpetuals conceptually. None connected the stopping-time structure to Islamic finance or to reduced-form credit modeling at $r = 0$.

Sukuk and takaful pricing literature. Jobst [36] and Jobst et al. [37] document the economics and issuance of Islamic securitization. Uddin et al. [54] document the LIBOR-embedding problem in sukuk pricing. Empirical studies consistently find that sukuk spreads behave similarly to conventional bond spreads (Reboredo et al. [49]; Hassan et al. [26]; Cakir & Raei [15]), meaning that the same mathematical credit risk machinery drives both — but that machinery currently requires $r > 0$. Our framework provides a riba-free alternative that preserves the same spread dynamics through κ. For takaful: Billah [10, 11] identifies the tension between actuarial fairness and riba-free discounting. Htay & Salman [27] attempt a riba-free contribution formula but encounter a circularity that the κ-rate framework resolves exactly, while the accounting and governance treatment of Htay et al. [28] sets out the operational standards a transparent, on-chain takaful pool must meet. The retakaful literature [2] develops contribution optimization models that are compatible with the present framework.

Islamic finance, jurisprudence, and fintech. El-Gamal [22] provides the canonical economic analysis of riba. Iqbal & Mirakhor [31] and Chapra [16] establish the normative framework. Khan & Ahmed [42] document that Islamic institutions are structurally underhedged. Kamali [39] provides the jurisprudential analysis of derivatives. Recent literature has begun exploring blockchain-based Islamic finance: Meera & Larbani [45] critique the ownership effects of fractional-reserve money creation from an Islamic perspective; more recently, tokenized sukuk on Ethereum [41] demonstrate that smart contracts can enforce the asset-backing requirement of AAOIFI standards at the protocol layer — directly enabling the on-chain κ-parameterized sukuk we propose in Section 6.1. SeekersGuidance [51] provides contemporary conservative jurisprudential support for the narrower claim that the $\iota = 0$ condition removes the *riba* objection specifically; the same ruling nonetheless maintains that leveraged perpetual contracts are impermissible on separate *gharar* and ownership (*qabdh*) grounds, which concern contractual structure rather than the pricing parameter.

Contributions

This paper makes seven contributions.

1. **Formal equivalence theorem.** We prove that the Ackerer-Hugonnier-Jermann perpetual pricing formula and the Duffie-Singleton defaultable bond formula are formally isomorphic (Proposition 1), and that no-arbitrage at $\iota = 0$ implies no-arbitrage for credit pricing at $r = 0$ (Corollary 1).
2. **Islamic perpetual sukuk.** We derive a closed-form no-arbitrage price for an Islamic perpetual sukuk using the κ-parameterized stopping time (Section 6.1).
3. **Actuarially fair takaful formula.** We apply Proposition 6 of Ackerer et al. to derive a closed-form everlasting takaful contribution pricing formula — riba-free and on-chain verifiable, with product-layer questions reserved to Shariah scholars (Section 6.2).
4. **Islamic credit default swap.** We derive a riba-free iCDS whose par spread equals $\kappa(1-\delta)$ and is independent of any interest rate, giving Islamic finance a credit-hedging instrument it has lacked (Section 6.3).
5. **The κ-rate.** We propose the convergence intensity κ as an Islamic alternative to the risk-free rate r, providing the first mathematically rigorous pricing foundation for a complete riba-free credit market (Section 7).
6. **Separation Theorem.** We formally establish that interest is an eliminable component of no-arbitrage pricing — the price at $\iota = 0$ is well-defined and arbitrage-free (Theorem 2) — providing the mathematical statement that Islamic finance has required but lacked.
7. **Shariah analysis.** We analyze the riba question formally and set out the gharar, maysir, and AAOIFI considerations [1] that remain for scholarly determination (Section 8).

2 The Ackerer-Hugonnier-Jermann Framework

2.1 Market Setup

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space with filtration $(\mathcal{F}_t)_{t\geq 0}$ satisfying the usual conditions (Protter [48]). There are two funding accounts: a long account growing at rate $r_a(t)$ and a short account growing at rate $r_b(t)$, both strictly positive. All pricing below is under the long's pricing measure Q^a, the risk-neutral (equivalent martingale) measure whose existence is equivalent to absence of arbitrage in the sense of Harrison & Kreps [25]. Let X_t denote the price of a spot asset satisfying, under Q^a:

$$\frac{dX_t}{X_t} = (r_a(t) - r_b(t))\,dt + \sigma_X\,dW_t^{Q^a} \tag{4}$$

where W^{Q^a} is a standard Brownian motion under Q^a and $\sigma_X > 0$ is the spot volatility. A perpetual futures contract on X_t with convergence parameter $\kappa > 0$ and interest parameter $\iota \geq 0$ is a continuously-settled contract with no fixed expiry.

2.2 The Perpetual Futures Price

Under constant parameters $r_a, r_b, \kappa, \iota, \sigma_X$, Proposition 3 of Ackerer et al. [3] establishes that the unique no-arbitrage perpetual futures price is:

$$f_t = \frac{\kappa - \iota}{\kappa + r_b - r_a} \cdot X_t \tag{5}$$

provided $\kappa + r_b - r_a > 0$. The ratio f_t/X_t is the perpetual *basis*. When $r_a = r_b$ (symmetric funding rates):

$$f_t = \frac{\kappa - \iota}{\kappa} \cdot X_t \tag{6}$$

2.3 The Random Time Representation

The central result of Ackerer et al. is a *random time representation* that reveals the economic structure of perpetual pricing. Define the **convergence stopping time** θ_t as a random variable with conditional density under Q^a:

$$\begin{aligned} q(\theta_t \in ds \mid \mathcal{F}_t) &= (\kappa_s - \iota_s) \\ &\quad \times \exp\left(-\int_t^s (\kappa_u - \iota_u)\,du\right) ds, \quad s \geq t \end{aligned} \tag{7}$$

This is a Cox process (doubly stochastic Poisson process) with intensity $\kappa_t - \iota_t$, as formalized in the theory of point processes by Brémaud [14] and the credit risk context by Lando [43]. The distribution of θ_t is exponential with rate $\kappa - \iota$ in the constant-parameter case:

$$Q^a(\theta_t > s \mid \mathcal{F}_t) = e^{-(\kappa-\iota)(s-t)}, \quad s \geq t \tag{8}$$

The perpetual futures price then satisfies (Theorem 3, Ackerer et al.):

$$f_t = \mathbb{E}_t^{Q^a}\left[\mathrm{e}^{-\int_t^{\theta_t} \iota_s\,ds} \cdot X_{\theta_t}\right] \tag{9}$$

The price is the discounted expected spot value at the random stopping time θ_t. The discount is ι (the interest parameter), not r_a or r_b.

2.4 The Critical Case: $\iota = 0$

When $\iota = 0$, Theorem 3 of Ackerer et al. establishes that no-arbitrage is satisfied. Substituting into equations (7)–(9):

$$q(\theta_t \in ds \mid \mathcal{F}_t) = \kappa\, e^{-\kappa(s-t)}\, ds \tag{10}$$

$$f_t = \mathbb{E}_t^{Q^a}[X_{\theta_t}], \quad \theta_t \sim t + \mathrm{Exp}(\kappa) \tag{11}$$

Three consequences follow immediately:

1. **No discounting.** The expectation in (11) is taken without any interest discount. The price is the pure expected value.
2. **κ carries all timing information.** The probability distribution of *when* payoff occurs is governed entirely by κ. Larger κ means faster convergence and shorter expected stopping time; smaller κ means slower convergence.
3. **The price formula at $r_a = r_b$.** From (6) with $\iota = 0$: $f_t = X_t$. The perpetual futures price equals the spot price identically — no premium, no discount — consistent with the empirical finding that dYdX v4 funding at $\iota = 0$ oscillates through zero (negative in ∼26% of intervals) with no structural floor, rather than pinning to an interest constant as $\iota > 0$ venues do [5]. (The *realised* sample mean over the bullish 2025 window was positive, ≈7%/yr annualised — a market premium, not a structural charge; the companion paper reports both numbers.)

Remark 1. The CEX interest parameter $\iota = 0.0001$ per 8 hours (= 10.95 %/year on Binance, Bybit, OKX) creates a permanent, market-independent wedge in equation (6): $f_t = \frac{\kappa-\iota}{\kappa} X_t \neq X_t$ whenever $\iota \neq 0$. This fixed, predetermined deviation of the perpetual from spot — present regardless of its sign — is structurally *riba al-fadl* (predetermined excess in exchange) under all four Sunni schools. Setting $\iota = 0$ eliminates it exactly ($f_t = X_t$).

3 Reduced-Form Credit Risk Models

3.1 The Duffie-Singleton Framework

The reduced-form approach to credit risk, initiated by Jarrow & Turnbull [33] and systematized by Duffie & Singleton [19, 20], models default as the first jump of a Cox process with stochastic intensity. The comprehensive mathematical treatment is provided by Bielecki & Rutkowski [9] and Jeanblanc, Yor & Chesney [35].

Let $(\Omega, \mathcal{F}, Q)$ be a risk-neutral probability space enlarged to carry the default indicator $H_t = \mathbf{1}_{\tau \le t}$ (Jeanblanc & Rutkowski [34]; Elliott, Jeanblanc & Yor [23]). Define the **default time** τ as a stopping time with respect to $(\mathcal{F}_t)$, characterized by the Q-conditional survival probability:

$$Q(\tau > s \mid \mathcal{F}_t) = \exp\left(-\int_t^s \lambda_u\, du\right), \quad s \ge t \tag{12}$$

where $\lambda_t > 0$ is the **hazard rate** (default intensity). The hazard rate has the conditional density:

$$q(\tau \in ds \mid \mathcal{F}_t) = \lambda_s \exp\left(-\int_t^s \lambda_u\, du\right) ds, \quad s \ge t \tag{13}$$

3.2 Defaultable Instrument Pricing

For a defaultable instrument with payoff function $\phi(X_\tau)$ at the default time τ and risk-free discount rate r_t:

$$P_t = \mathbb{E}_t^Q\left[\mathrm{e}^{-\int_t^\tau r_s\, ds} \cdot \phi(X_\tau)\right] \tag{14}$$

In the constant-parameter case (r, λ constant, $\tau \sim t + \mathrm{Exp}(\lambda)$), this gives:

$$P_t = \int_t^\infty e^{-r(s-t)}\ \mathbb{E}_t[\phi(X_s)]\ \lambda e^{-\lambda(s-t)}\, ds \tag{15}$$

which admits the closed form $P_t = \frac{\lambda}{r+\lambda}\,\phi(X_t)$ when ϕ is linear and the spot is a Q-martingale. For a bond with full recovery ($\phi(x) = x$) and Q-martingale spot:

$$P_t = \frac{\lambda}{r+\lambda} \cdot X_t \tag{16}$$

In the conventional framework, $r > 0$ is what makes $P_t < X_t$: with no interest discount the bond price equals the spot, seemingly erasing the credit spread. Section 4 resolves this.

4 The Mathematical Equivalence

4.1 The Isomorphism

The resolution begins by observing that the credit and perpetual frameworks share a single mathematical skeleton. Comparing equations (7) and (13) reveals an immediate structural identity:

AHJ Perpetuals	**Duffie-Singleton Credit**	**Role**
θ_t	τ	Random stopping time
$\kappa_t - \iota_t$	λ_t	Event intensity*
ι_t	r_t	Discount parameter
X_{θ_t}	$\phi(X_\tau)$	Payoff at event
f_t	P_t	Instrument price
Q^a	Q	Pricing measure

*The AHJ stopping intensity is $\kappa - \iota$ (equation (7)); at the riba-free point $\iota = 0$ it coincides with κ, giving the headline identification $\kappa \leftrightarrow \lambda$ used throughout.

The two frameworks share an identical mathematical skeleton. The perpetual futures framework uses κ and ι; the credit framework uses λ and r. The functions they play are identical.

4.2 The Main Result

Proposition 1 (Random Stopping Time Equivalence)**.** *Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a filtered probability space with the usual conditions. Let X_t be an asset price process satisfying (4) under a pricing measure $Q = Q^a$. Define:*

1. *A perpetual futures contract with parameters κ, ι, priced by equation (9).*
2. *A defaultable financial claim with hazard rate λ, discount rate r, and recovery $\phi(X_\tau)$, priced by equation (14).*

Under the exact parameter identification $\kappa - \iota \leftrightarrow \lambda$, $\iota \leftrightarrow r$, $X_{\theta_t} \leftrightarrow \phi(X_\tau)$:

$$f_t = P_t \tag{17}$$

The two pricing formulas are formally isomorphic for every $\iota \geq 0$. At the riba-free point $\iota = r = 0$ the identification reduces to the headline form $\kappa \leftrightarrow \lambda$ used throughout this paper.

Proof. The stopping time densities (7) and (13) are identical under $\kappa - \iota \leftrightarrow \lambda$: by (7) the AHJ stopping intensity is $\kappa - \iota$, which maps to the credit intensity λ. The integral representations (9) and (14) are then identical under the additional identification $\iota \leftrightarrow r$ and $X_{\theta_t} \leftrightarrow \phi(X_\tau)$. The pricing measures Q^a and Q are both risk-neutral with respect to their respective numeraires. The formal identity (17) follows directly, and at $\iota = 0$ the intensity identification is $\kappa \leftrightarrow \lambda$. □

Corollary 1 (Riba-Free Credit Pricing is No-Arbitrage)**.** *Setting $\iota = 0$ in the AHJ perpetual futures framework yields a no-arbitrage price by Theorem 3 of [3]. Under the isomorphism of Proposition 1, the corresponding credit instrument at $r = 0$ is also no-arbitrage, with pricing formula:*

$$\boxed{P_t = \mathbb{E}_t^Q[\phi(X_\tau)], \qquad \tau \sim t + \mathrm{Exp}(\kappa), \quad r = 0} \tag{18}$$

The hazard rate κ alone carries all timing information. No positive interest rate is required.

Proof. By Theorem 3 of Ackerer et al. [3], setting $\iota = 0$ satisfies the no-arbitrage conditions for the perpetual futures. By Proposition 1, $f_t = P_t$ under the given identification. Therefore P_t at $r = 0$ satisfies the same no-arbitrage conditions. Specifically: $P_t = \mathbb{E}_t^Q[\phi(X_\tau)]$ is arbitrage-free, being the expectation of the undiscounted payoff under an equivalent martingale measure with Q-intensity κ. □

Remark 2. In this paper's applications (sukuk redemption, takaful triggers) the stopping time τ is contractually specified, mirroring AHJ's contractual stopping time, so the perpetual-style uniqueness argument applies directly. For exogenous default (the iCDS of Section 6.3) the Q-intensity is an input calibrated from market spreads, as in conventional reduced-form practice; the market is incomplete and the intensity is not identified by replication in the underlying alone.

4.3 Resolving the Apparent Paradox

Corollary 1 may appear paradoxical: how can a credit instrument with $r = 0$ have $P_t < X_t$ (i.e., trade at a discount) without interest? The resolution requires care, because two distinct instruments are in play and they behave differently in κ. The discount-from-par comes from the recovery haircut and is κ-independent; the per-annum *spread* comes from a fixed-maturity survival claim, in which κ does enter. We treat them in turn.

From equation (6) with $\iota = 0$:

$$\frac{f_t}{X_t} = 1 \qquad (r_a = r_b,\ \iota = 0) \tag{19}$$

First, consider a claim that pays the recovery value $\phi(X_\tau) = \delta \cdot X_\tau$ at the default time τ, where $\delta \in (0, 1)$ is the recovery fraction. Since X is a Q-martingale and τ is a stopping time, optional stopping gives $\mathbb{E}_t^Q[X_\tau] = X_t$, so

$$P_t = \delta \cdot \mathbb{E}_t^Q[X_\tau] = \delta \cdot X_t < X_t. \tag{20}$$

The discount from par is $1 - \delta$: the credit loss on default. Note that κ *does not appear* here — because the payoff is collected *at* the random default time, the timing intensity cancels, and the price is the pure recovery haircut. This is not interest; it is the recovery-rate pricing of credit risk.

The per-annum credit *spread* arises in a different instrument: a claim of fixed maturity T that pays par at T if no default occurs by T, and whose default recovery is a fraction δ of its pre-default market value — the recovery-of-market-value convention of Duffie & Singleton [19]. At $r = 0$ its price is exactly $P_t = \mathcal{P}\, e^{-\kappa(1-\delta)(T-t)}$, where $\mathcal{P}$ is the par value, giving a continuously-compounded spread of exactly $\kappa(1 - \delta)$. (Under recovery of face the $r = 0$ price is instead $\mathcal{P}\,[\delta + (1-\delta)e^{-\kappa(T-t)}]$, whose spread equals $\kappa(1 - \delta)$ to first order in $\kappa(T - t)$.) Here κ enters directly. Both instruments are priced with $\iota = 0$ and neither uses an interest rate: the first isolates the recovery haircut, the second the hazard-driven spread. This is the separation of credit risk from time preference that Islamic finance requires.

Remark 3. In the conventional framework, the credit spread over the risk-free rate bundles two economically distinct risks: (1) the probability of default (hazard rate λ) and (2) the time value of money (risk-free rate r). The Ackerer framework separates them definitively. At $\iota = r = 0$, only the hazard rate κ remains, representing pure credit risk with no interest component.

4.4 Empirical Evidence: Negative Interest Rate Environments

Our theoretical result — that credit markets can price correctly at $r = 0$ — finds empirical corroboration in the European negative interest rate experiment (2014–2022). The European Central Bank reduced its Deposit Facility Rate to -0.10% in June 2014, eventually reaching -0.50% by September 2019 (ECB [24]).

During this period, CDS markets and corporate bond markets continued to function efficiently in the eurozone: credit spreads reflected default risk accurately; sukuk issuance continued (IIFM [29]); and institutional credit pricing did not break down. The absence of a positive r in some jurisdictions did not render credit derivatives unpriceable.

This is precisely the empirical analog of our theoretical result: the credit pricing machinery (hazard rates, survival probabilities, recovery values) does not require $r > 0$ for mathematical coherence. The interest rate is indeed eliminable from credit pricing, as our Separation Theorem (Theorem 2) shows. At $r \approx 0$ or $r < 0$, markets revealed empirically what the $\iota = r = 0$ correspondence establishes theoretically.

4.5 Direct Validation: $\hat{\kappa} = s/(1-\delta)$ on Real Sukuk

The negative-rate episode shows credit markets functioning without a positive r; we can go further and test the central identification of this paper — the recovery-discount relation $s = \kappa(1-\delta)$ of Section 4, equivalently $\hat{\kappa} = s/(1-\delta)$ — *directly* on observed sukuk. Two Cbonds pulls are used: a dedicated sovereign-screener cross-section (61 sukuk from 12 issuers, including Bahrain, Egypt, Oman, Pakistan and Türkiye, with ratings down to CCC — the sample of Table 1), and the broader active-quote snapshot of **195 instruments** (40 sovereign, 154 corporate, one municipal), spanning investment-grade and, via the corporates, high-yield ratings. For each instrument we take the observed G-spread s over the maturity-matched US Treasury curve — which, for these bullet sukuk, closely approximates the option-adjusted spread — and apply the identification with a senior recovery $\delta = 0.60$ [47].

Two predictions of the theory hold on the real cross-section (Table 1). First, **magnitude**: the implied $\hat{\kappa}$ has median 0.02 per annum (sovereign subset 0.019) — exactly the 10^{-2} order of magnitude the credit interpretation of κ requires. It is read off real prices, not chosen as a free parameter. Second, **credit monotonicity**: $\hat{\kappa}$ separates cleanly across the investment-grade/high-yield boundary, from ≈ 0.01 per annum at single-A to ≈ 0.05–0.07 at B/CCC, confirming that κ — identified with *zero* interest input — is genuinely measuring credit intensity, precisely as the isomorphism $\kappa \leftrightarrow \lambda$ predicts. The Separation Theorem is thus not only provable but *measurable*: the credit spread is recovered as $\kappa(1-\delta)$ on traded instruments, with no interest term.

The recovery assumption only rescales the level: $\delta \in [0.4, 0.7]$ multiplies every $\hat{\kappa}$ by a common factor and leaves the ordering untouched. That the extracted $\hat{\kappa}$ is genuine credit risk, and not an artefact of Islamic contract structure, is confirmed by an instrument-level sukuk-versus-conventional comparison on the same issuers. Matched at maturity, the USD sukuk and conventional bonds of Indonesia, Saudi Arabia and Türkiye price within ≈ 1 bp (per-issuer median sukuk-minus-conventional G-spreads of -1.3, -0.9 and $+0.0$ bp; the pooled median across all 48 matched pairs, which also include Bahrain's -48 bp captive-Islamic-bid exception, is -2.4 bp) [6]. The sukuk spread is therefore the sovereign credit spread, and $\hat{\kappa} = s/(1-\delta)$ recovers the same hazard intensity λ the isomorphism identifies it with. The relation is dynamic as well as static: daily Cbonds histories of 54 sukuk from five issuers (2018–2026; a dedicated dynamics pull that also covers high-yield issuers outside the 195-bond active-quote snapshot), reconstructed at fixed maturity to remove roll-down, show κ mean-reverting to issuer-specific long-run levels (investment grade ≈ 1–3%, high yield ≈ 5–7% per annum; cf. the sovereign-spread dynamics literature [21]), with the Feller non-negativity condition $2\alpha\bar{\kappa} \geq \nu^2$ satisfied in every series — the empirical counterpart of the stochastic-κ extension of Section 7.3.

Rating	Median s (bp)	Median $\hat{\kappa}$ (yr^{-1})
AA (IG)	72	0.018
A (IG)	37	0.009
BBB (IG)	78	0.019
BB (HY)	195	0.049
B (HY)	215	0.054
CCC (HY)	295	0.074

Table 1: The identification $\hat{\kappa} = s/(1-\delta)$ on the real sovereign USD-sukuk cross-section (dedicated sovereign-screener Cbonds pull, 61 bonds, 12 issuers; $\delta = 0.60$; s = observed G-spread over Treasury). $\hat{\kappa}$ sits in the predicted 10^{-2} per-annum credit regime and rises across the IG/HY boundary (the AA bucket, Saudi-dominated, prices just above single-A — the one investment-grade inversion); the separate 195-instrument active-quote snapshot (sovereign + corporate; its filter excludes the high-yield sovereigns shown here) gives the same median magnitude (0.02) and the same broadly-increasing credit ordering. Full instrument-level analysis in the companion κ-yield-curve paper [6].

5 Everlasting Options and the Takaful Pricing Formula

Proposition 6 of Ackerer et al. [3] extends the random time framework to arbitrary payoff functions $\phi(X_\tau)$, yielding closed-form **everlasting option** prices. Unlike the classical fixed-expiry no-arbitrage option price of Black & Scholes [13], which discounts the terminal payoff at the risk-free rate r over a deterministic horizon, the everlasting option prices the payoff at a *random* stopping time and, at $\iota = 0$, carries no interest discount at all. This extension is the key to takaful pricing.

5.1 The Everlasting Put Option

For a payoff $\phi(x) = \max(K - x, 0)$ (floor protection at strike K) with $\iota = 0$ and $r_a = r_b$, the everlasting put is the no-arbitrage value of the floor payoff delivered at the random convergence time $\tau \sim t + \mathrm{Exp}(\kappa)$ (Proposition 6 of Ackerer et al. [3]):

$$\Pi_{\text{put}}(x,K) = \mathbb{E}^{Q^a}\left[(K - X_\tau)^+\right]. \tag{21}$$

Let $\beta_p < 0 < 1 < \beta_c$ be the two roots of the characteristic equation

$$\frac{\sigma_X^2}{2}\,\beta(\beta-1) - \kappa = 0 \qquad (\iota = 0,\ r_a = r_b), \tag{22}$$

that is,

$$\beta_p = \tfrac{1}{2} - \sqrt{\tfrac{1}{4} + \tfrac{2\kappa}{\sigma_X^2}}, \qquad \beta_c = \tfrac{1}{2} + \sqrt{\tfrac{1}{4} + \tfrac{2\kappa}{\sigma_X^2}}, \qquad \beta_p + \beta_c = 1. \tag{23}$$

Because the genuine expectation (21) is C^1 and carries intrinsic value when in the money, the everlasting put is the *two-region* solution of Proposition 6 (value-matching and smooth-pasting at $x = K$, with the intrinsic value $K - x$ as particular solution where $x < K$):

$$\Pi_{\text{put}}(x,K) = \begin{cases} C_p\, x^{\beta_p}, & x \ge K, \\ (K - x) + \dfrac{K^{1-\beta_c}}{\beta_c - \beta_p}\, x^{\beta_c}, & x < K, \end{cases} \qquad C_p = \frac{K^{1-\beta_p}}{\beta_c - \beta_p}. \tag{24}$$

The coefficient denominator is $\beta_c - \beta_p = 2\sqrt{\frac{1}{4} + 2\kappa/\sigma_X^2}$ (the gap between the two roots), not $1 - \beta_p$: a single power law $C_p x^{\beta_p}$ extended through $x < K$ would omit the intrinsic value and misprice the floor.

5.2 The Everlasting Call Option

For $\phi(x) = \max(x - K, 0)$, the everlasting call is, symmetrically,

$$\Pi_{\text{call}}(x,K) = \begin{cases} \dfrac{K^{1-\beta_c}}{\beta_c - \beta_p}\, x^{\beta_c}, & x \le K, \\ (x - K) + C_p\, x^{\beta_p}, & x > K, \end{cases} \tag{25}$$

with the same root gap $\beta_c - \beta_p$ in every coefficient. With these (AHJ Proposition 6) coefficients the everlasting put–call parity holds exactly, $\Pi_{\text{call}} - \Pi_{\text{put}} = X_t - K$ (forward parity without discounting, since $\mathbb{E}^{Q^a}[X_\tau] = X_t$); a single power law in each leg would violate it.

6 Applications in Islamic Finance

6.1 Islamic Perpetual Sukuk

Sukuk spreads have been extensively documented in empirical literature [49, 26, 15]: they behave like credit spreads, with ratings and asset quality as primary drivers. The existing pricing models, however, embed r (LIBOR/SOFR) at every step [54, 37]. The framework below preserves the spread dynamics through κ while removing riba.

Definition 1 (Islamic Perpetual Sukuk)**.** An Islamic Perpetual Sukuk is a financial instrument in which:

1. The sukuk holder contributes capital backed by a real asset (RWA) with market value X_t.
2. The issuer (asset operator) pays a continuous premium $\pi_t = \kappa(f_t - X_t)\,dt$ to the holder — the perpetual futures funding rate under the κ-parameterized formula.
3. At a random redemption time τ (oracle-triggered by asset completion, sale, or Shariah board review), the holder receives the fair market value X_τ of the underlying asset.
4. The interest parameter $\iota = 0$ at all times.

Remark 4. The random redemption time τ is analogous to the "purchase undertaking" (*wa'd*) mechanism already approved under AAOIFI Standard 17 [1, 4]. Smart-contract implementation on a public blockchain makes the redemption trigger transparent and independently verifiable, eliminating the opacity (*gharar*) that traditional wa'd structures carry [41].

Proposition 2 (Sukuk No-Arbitrage Price)**.** *The fair price of the Islamic Perpetual Sukuk at time t is:*

$$P_t^{\text{sukuk}} = \mathbb{E}_t^Q[X_\tau] = \frac{\kappa}{\kappa + r_b - r_a} \cdot X_t \tag{26}$$

with $r_a = r_b$ (symmetric bilateral rates) giving $P_t = X_t$. This price is no-arbitrage by Corollary 1.

Proof. The redemption time $\tau \sim t + \mathrm{Exp}(\kappa)$ under the pricing measure Q. With recovery $\phi(X_\tau) = X_\tau$, Corollary 1 gives $P_t = \mathbb{E}_t^Q[X_\tau]$. Under the geometric Brownian motion (4): $\mathbb{E}_t^Q[X_\tau] = X_t \cdot \kappa/(\kappa + r_b - r_a)$, which is equation (26). No-arbitrage follows from Corollary 1. □

Economic interpretation. The sukuk is priced at the expected asset value at redemption, with no interest discount. The premium payments π_t transfer the basis risk from the issuer to the holder: when the asset trades at a premium to spot ($f_t > X_t$), the operator pays the holder; when at a discount, the holder pays the operator. This is risk-sharing (*mudarabah*-compatible) rather than a fixed return on capital.

The demand-side question, stated plainly. An honest reading of the constant-parameter equilibrium should be recorded here rather than left for a referee to find. At $r_a = r_b$ and $\iota = 0$ the model gives $f_t = X_t$ identically, so the equilibrium premium flow $\pi_t = \kappa(f_t - X_t)\,dt$ is *zero*: the holder pays X_t, receives X_τ at redemption, and earns nothing in expectation beyond exposure to the asset itself. That is exactly what an interest-free instrument *should* deliver in this benchmark — the programme's monetary papers show the economy prices time through state-contingent equity returns, not through this instrument — but it poses a real product-design question: why hold the perpetual sukuk rather than the asset? The answers live outside the constant-parameter benchmark (liquidity and divisibility of the claim versus the asset; the funding flow when $r_a \neq r_b$ or when the basis deviates;

monetary and regulatory roles of a standardised claim), and the welfare value of what the instrument does *not* provide — a riskless intertemporal trade — is quantified in the companion safe-asset paper. We flag the question because a pricing theorem is not a demand theory, and the corpus should not read as if it were.

The κ parameter for sukuk. Two distinct intensities must not be conflated. For the perpetual sukuk above, κ is the contractual *redemption intensity* ($\kappa_{\text{red}} = 1/\text{expected horizon}$): a sukuk backed by an infrastructure asset with a 5-year expected horizon gives $\kappa_{\text{red}} \approx 0.20$; a 10-year asset, $\kappa_{\text{red}} \approx 0.10$. Credit quality — rating, asset quality, macro conditions [49, 26] — enters through the priced default-intensity component: the credit $\hat{\kappa} = s/(1-\delta) \sim 10^{-2}$ per annum of Section 4.5, not through $1/T$. Both are directly observable and estimable without any interest rate input.

6.2 Takaful: Actuarially Fair Perpetual Insurance

Definition 2 (Everlasting Takaful Contract)**.** An Everlasting Takaful Contract is a mutual insurance arrangement in which:

1. Participants pay a continuous premium $\pi\,dt$ per unit time into a mutual pool until the claim event τ.
2. The pool pays $\phi(X_\tau) = \max(K - X_\tau, 0)$ — the shortfall of asset value below the insured floor K — at the claim event τ.
3. The claim event follows a Cox process with intensity κ (the historical hazard rate of the insured event: drought, flood, death, property damage).
4. No investment return is generated on the pool; it earns at rate $\iota = 0$.

Theorem 1 (Actuarially Fair Takaful Premium)**.** *Under actuarial equilibrium (PV of contributions = PV of expected claim), the fair continuous premium rate π and the lump-sum contribution Π at inception for floor protection at level K, given current asset value $x = X_t$, intensity κ, and volatility σ_X, satisfy:*

$$\Pi_{\text{takaful}}(x, K) = \frac{K^{1-\beta_p}}{\beta_c - \beta_p} \cdot x^{\beta_p} \qquad (x \geq K) \tag{27}$$

$$\pi_{\text{continuous}} = \kappa \cdot \Pi_{\text{takaful}}(x, K) \tag{28}$$

where $\beta_p = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{2\kappa}{\sigma_X^2}} < 0$. Both prices are no-arbitrage (Corollary 1) and require no positive interest rate.

Proof. Lump-sum contribution. By Corollary 1, the no-arbitrage price of $(K - X_\tau)^+$ at $r = 0$ is $\mathbb{E}_t^Q[(K - X_\tau)^+]$. By Proposition 6 of Ackerer et al. [3] applied with $\iota = 0$ and $r_a = r_b$, this equals $C_p x^{\beta_p}$ with C_p, β_p as in (24) and (23), giving (27).

Continuous premium. If the participant pays continuous premium π until $\tau \sim t + \text{Exp}(\kappa)$, the present value of premium flows at $r = 0$ is $\mathbb{E}[\int_t^\tau \pi\, ds] = \pi/\kappa$. Setting $\pi/\kappa = \Pi_{\text{takaful}}$ gives $\pi = \kappa \cdot \Pi_{\text{takaful}}$, which is (28). $\square$

Physical vs. pricing intensity. The definition specifies κ as the historical hazard rate (a physical-measure object), while Theorem 1 prices under Q; the actuarially fair premium equals the no-arbitrage price only if the claim intensity carries no risk premium, i.e. $\kappa^P = \kappa^Q$. This identification is appropriate for idiosyncratic, diversifiable claims (individual mortality, property damage); for systematic perils that strike many participants simultaneously (regional drought or flood), the pricing intensity κ^Q embeds an event-risk premium and must be calibrated separately from the historical hazard. This distinction governs the whole program and deserves a taxonomy: *spread- and price-implied* intensities (the sovereign and corporate $\hat{\kappa}$ of the companion yield-curve study) are Q-measure objects — the credit-spread literature finds such spreads run a multiple of actuarial expected loss — while *frequency-based* intensities (crop-loss, mortality) are P-measure objects. The two are commensurable in *kind* (both are event intensities, neither a price of time — the Shariah-relevant property) but not in *level*; downstream constructions that mix them (portfolio floors, cross-domain comparisons) should be read as conservative whenever Q-volatilities exceed P-volatilities, and takaful priced at κ^P is the actuarially fair point for a mutual pool precisely because no event-risk premium is extracted from participants.

Agricultural takaful example. Let X_t be the market price of wheat (USD/bushel). A farmer participates in a mutual takaful pool offering floor protection at $K =$ \$6.00/bushel. Historical drought frequency: $\kappa = 0.08$ (one event every ∼12.5 years). Wheat volatility: $\sigma_X = 0.25$ (25% p.a.). Current wheat price: $x = \$7.50$.

$$\begin{aligned}
\beta_p &= 0.5 - \sqrt{0.25 + 2.56} = -1.176, \quad \beta_c = 0.5 + 1.676 = 2.176 \\
C_p &= \frac{(6.00)^{2.176}}{\beta_c - \beta_p} = \frac{49.35}{3.352} = 14.72 \\
\Pi &= 14.72 \times (7.50)^{-1.176} \\
&= 14.72 \times 0.0935 = \$1.38/\text{bushel}
\end{aligned} \tag{29}$$

$$\begin{aligned}
\pi_{\text{continuous}} &= 0.08 \times 1.38 \\
&= \$0.110/\text{bushel/year}
\end{aligned}$$

The farmer contributes \$1.38 per bushel at inception (or \$0.110 per year continuously). This is actuarially fair. No interest rate was used in any step.

A design caveat this example must carry. The Cox construction models the claim event τ and the price X_t as *independent*, and for this peril they are not: a drought is a supply shock that tends to *raise* the crop price, so a price-floor payoff $(K - X_\tau)^+$ evaluated at the drought time pays *least* exactly when the insured event strikes — while the farmer's actual loss is in *quantity*, not price. As specified, the contract is therefore floor protection against *low-price* states (a bumper-harvest or demand-collapse hedge), not drought indemnity; a drought product should pay on a *yield* index (as the companion takaful paper's loss-cost calibration does) or model the event–price correlation explicitly, at the cost of the closed form. The mathematics of the example is exact; its peril label is the thing to design carefully. Life takaful, next, is immune to this issue — the benefit is a fixed B, not a function of a correlated price.

Life takaful — continuous premium formula. For a fixed death benefit B payable at death $\tau \sim \mathrm{Exp}(\kappa_{\text{mort}})$ (where κ_{mort} is the age-adjusted mortality hazard rate from actuarial tables), the fair continuous premium π_{life} satisfies:

$$\frac{\pi_{\text{life}}}{\kappa_{\text{mort}}} = B \quad \Longrightarrow \quad \pi_{\text{life}} = \kappa_{\text{mort}} \cdot B \tag{30}$$

The participant pays $\kappa_{\text{mort}} B$ per unit time until death; the pool pays B at death. Both sides discount at $r = 0$. A 35-year-old with $\kappa_{\text{mort}} \approx 0.002$ and $B = \$100{,}000$ pays \$200/year — consistent with real-world life takaful contributions for that age bracket. As the participant ages (κ_{mort} rises), the formula automatically reprices contributions upward, with full on-chain transparency.

6.3 Islamic Credit Default Swaps (iCDS)

The global CDS market has a notional outstanding of approximately \$10 trillion [8], built entirely on the Duffie-Singleton framework with $r > 0$. Islamic institutions have no equivalent instrument for credit risk hedging [42].

Definition 3 (Islamic CDS (iCDS))**.** An iCDS is a bilateral contract in which:

1. The protection buyer pays a continuous premium π_{iCDS} per unit time to the protection seller.
2. At the credit event time τ (default, restructuring, or rating trigger), the seller pays the loss given default $\mathrm{LGD}_\tau = (1-\delta)X_\tau$ to the buyer, where $\delta \in [0, 1]$ is the recovery rate.
3. The interest parameter $\iota = 0$; no fixed rate premium.

Proposition 3 (iCDS Fair Spread)**.** *The fair iCDS continuous premium (the* iCDS spread*)* s^* *when the reference sukuk has recovery rate* δ*, current value* X_t*, default intensity* κ*, and* $\iota = 0$ *is:*

$$s^* = \kappa\,(1 - \delta) \tag{31}$$

This is the continuous premium rate that equates the PV of protection received to the PV of premiums paid.

Proof. PV of protection received $= \mathbb{E}[(1-\delta)X_\tau] = (1-\delta)X_t$ (by Corollary 1 with $\phi(x) = (1-\delta)x$). PV of premiums paid $= \mathbb{E}[\int_t^\tau s^*\, ds] = s^*/\kappa$. Setting equal: $s^*/\kappa = (1-\delta)X_t/X_t = 1-\delta$ (normalized to par $X_t = 1$), giving $s^* = \kappa(1-\delta)$. □

The iCDS spread $s^* = \kappa(1-\delta)$ has the correct economic properties: it increases with κ (higher default risk) and decreases with δ (higher recovery). This is structurally identical to the conventional CDS spread $s = \lambda(1-\delta)$, with κ replacing λ and no r appearing anywhere.

Pricing intensity and the substance-compliant form. Removing ι removes the price of time, but Proposition 3 as stated prices at the market intensity κ, which — by the taxonomy of §6.2 — is the Q-measure, spread-implied object κ^Q: it embeds the event-risk premium the credit-spread literature finds to run a multiple of actuarial expected loss. Whether charging that premium is itself permissible, once every predetermined charge is gone, is the substantive question the corpus reserves. There is, however, a form that does not raise it. Structured as a *cooperative guarantee* — a *kafala*/*tabarru* pool in which members mutually indemnify realised default loss, gated on insurable interest and possession of the reference obligation — the iCDS contribution is set at the realised hazard κ^P,

$$s^*_{\text{coop}} = \kappa^P\,(1 - \delta), \tag{32}$$

the actuarially fair expected loss, exactly as the mutual-pool contribution of §6.2 is set at κ^P "precisely because no event-risk premium is extracted from participants." The contested wedge $(\kappa^Q - \kappa^P)(1-\delta)$ is then not *sold* to a protection writer for profit but retained by the pool and returned as surplus, net of a disclosed *wakala* administration fee. So priced, credit-loss cover is structurally the takaful case this paper has already shown to be actuarially fair and free of both riba and the event-risk premium — collapsing the credit case into the insurance case that needs no further resolution, and leaving only the narrower, shared questions of whether tokenised title constitutes valid possession and whether the *wakala* loading is a genuine fee.

6.4 Sovereign Perpetual Sukuk

Sovereign governments in Muslim-majority countries (Malaysia, Saudi Arabia, Indonesia, Türkiye, Pakistan) issue sukuk to fund infrastructure. These are priced against SOFR or government bond yields, embedding riba implicitly [29, 32]. The Islamic perpetual sukuk framework resolves this.

Example 1 (Infrastructure Perpetual Sukuk)**.** A sovereign issues a perpetual sukuk backed by a highway project with expected completion in 7 years: the redemption intensity is $\kappa_{\text{red}} = 1/7 \approx 0.143$. The project asset value is X_t (updated quarterly by a government oracle). Under $r_a = r_b$ and $\iota = 0$: $P_t = X_t$ (at-par perpetual).

The sovereign pays premium $\pi_t = \kappa(f_t - X_t)\,dt$ continuously to sukuk holders. On completion (random event τ), holders receive X_τ. Secondary market price: $P_t = X_t$, updated via observed κ from reported completion timeline. *No yield-to-maturity calculation. No SOFR benchmark. No riba.*

The κ parameter functions as a *sovereign credibility signal*: stable infrastructure management exhibits high, predictable κ; governance failures widen the basis and imply declining κ — a market-based credit signal without any interest rate.

7 The κ-Rate: An Islamic Monetary Alternative

7.1 The Problem with r

The risk-free interest rate r is the fundamental pricing parameter of conventional finance: exogenous, set by central banks, and definitionally *riba* (it compensates capital for time elapsed, regardless of real economic outcome). Every discount factor in every asset pricing model contains r. Islamic finance has no alternative [16, 31].

7.2 The κ-Rate as a Substitute

Definition 4 (κ-Rate)**.** The κ-rate is the convergence intensity of a perpetual financial instrument linking a price claim to an underlying real asset. It satisfies:

1. **Endogenous determination.** κ is backed out from observed market prices:
$$\hat{\kappa}_t = \frac{f_t/X_t \cdot (r_b - r_a)}{1 - f_t/X_t} \tag{33}$$
valid when $r_a \neq r_b$. In the symmetric regime $r_a = r_b$ the basis is identically one and (33) is uninformative; κ is then identified from episodes of funding asymmetry, from the decay rate of transient basis deviations (whose convergence speed is κ by construction), or — as in Section 4.5 — directly from traded sukuk spreads via $\hat{\kappa} = s/(1-\delta)$. It reflects real economic convergence dynamics, not central bank policy.
2. **Observable and transparent.** κ is computable from any two market prices (f_t and X_t), publishable on-chain, and independently verifiable.
3. **No riba content.** κ is an event intensity, not a reward for the time value of money. It compensates for credit risk (sukuk), actuarial risk (takaful), or convergence risk (perpetuals).
4. **Reviewable by Shariah boards.** κ represents neither interest on capital nor predetermined excess, and is therefore submitted to Shariah-board review as an event intensity; product-level approval remains a jurisprudential determination.

7.3 Extensions: Stochastic κ and Affine Models

Definition 4 takes κ as constant. In practice, convergence intensities vary over time (as project timelines lengthen or credit conditions deteriorate). The affine credit risk framework of Duffie & Singleton [19, 20] and the general theory of Bielecki & Rutkowski [9] provide the natural extension.

Let κ_t follow a CIR process [17]:
$$d\kappa_t = \alpha(\bar{\kappa} - \kappa_t)\,dt + \nu\sqrt{\kappa_t}\,dZ_t \tag{34}$$
where $\bar{\kappa} > 0$ is the long-run mean intensity, $\alpha > 0$ is mean-reversion speed, and $\nu > 0$ is the vol-of-κ. Under this specification, the stopping time density (7) remains well-defined but the closed-form prices of Sections 6.1–6.4 require the Laplace transform of $\int_t^T \kappa_s\,ds$ over a fixed horizon T (available in closed form for the CIR specification via Jeanblanc, Yor & Chesney [35], Prop. 6.3.4):
$$Q(\tau > T \mid \mathcal{F}_t) = \mathbb{E}_t^Q\left[e^{-\int_t^T \kappa_s\,ds}\right] = A(T-t)\,e^{-B(T-t)\kappa_t} \tag{35}$$
where $A(\cdot)$ and $B(\cdot)$ are the standard CIR bond price functions, valid on the pre-default event $\{\tau > t\}$. All qualitative results — no-arbitrage, the Separation Theorem, riba-freedom at the pricing layer — hold under stochastic κ; only the closed-form expressions require numerical methods. This specification is also empirically grounded: as reported in Section 4.5, the κ backed out of real sovereign sukuk is observably mean-reverting toward issuer-specific long-run means and satisfies the CIR Feller condition $2\alpha\bar{\kappa} \geq \nu^2$ in every series we estimate, so (34) describes the data rather than merely extending the theory.

7.4 The Redemption-Horizon Curve and the Credit κ-Yield Curve

Just as conventional finance builds a yield curve $r(T)$ as a function of maturity T, Islamic finance can build a **redemption-horizon curve**: the mapping $\kappa_{\text{red}}(T)$ of contractual redemption intensities from perpetual sukuk of different expected maturities.
$$\kappa_{\text{red}}(T) = \frac{1}{T}, \qquad T = \text{expected horizon} \tag{36}$$
Derived from the exponential distribution ($\kappa = 1/E[\tau]$), the redemption-horizon curve is:

- Downward-sloping by construction (short assets have higher κ_{red}).
- Grounded in real project timelines, not monetary policy expectations.

The traded *credit* κ-yield curve — constructed in the companion paper [6] from observed sukuk spreads via $\hat{\kappa} = s/(1-\delta)$ — is a distinct, measured object and is mildly *upward*-sloping in maturity for investment-grade issuers (+1.17 to +3.4 bp/yr); together the two curves supply the term-structure inputs that the conventional yield curve supplies in Islamic finance contexts.

Instrument	**κ calibration**	**Data source**
Perpetual sukuk	Expected project timeline	Engineering reports
Takaful	Historical claim frequency	Actuarial tables
iCDS	Reference entity default history	Rating agencies
Sovereign sukuk	Project completion probability	Government data

8 Shariah Analysis

Following the jurisprudential boundary set out in the companion research note [7], the formal analysis below settles only the *riba* question at the pricing layer; gharar, maysir, and possession (qabd) are product-layer questions governed by rules R1–R4 of that note and reserved to Shariah scholars. The gharar, maysir, and AAOIFI material that follows is submitted as structural input to that review, not as a verdict.

8.1 Riba: Why $\iota = 0$ Satisfies All Schools

The prohibition of *riba* is universal across all four Sunni schools (Hanafi, Maliki, Shafi'i, Hanbali) and the Ja'fari (Shi'a) school [55, 22]. The Quranic prohibition is absolute (Al-Baqarah 2:275–279; Al-Imran 3:130).

Riba takes two forms:

1. *Riba al-fadl*: excess in exchange. The CEX interest floor $\iota > 0$ is precisely this: longs always pay more than the market premium, with the excess $(\iota \cdot f_t)$ predetermined and fixed.
2. *Riba al-nasi'ah*: excess for delay. The conventional risk-free rate r is this: the lender receives r per unit time regardless of economic outcome.

The κ-rate is neither. It is:

- Not predetermined: κ fluctuates with observed market conditions, backed out from market prices per (33).
- Not a reward for time: κ compensates for the probability and timing of a credit/claim/convergence event (real economic risk), not for the passage of time per se.
- Symmetric: the stopping time distribution does not systematically benefit either party.

Remark 5. SeekersGuidance (2025) — a rigorous contemporary conservative Shariah ruling on perpetual futures — concedes that removing the interest component addresses the *riba* objection, while ultimately maintaining that leveraged perpetuals are impermissible on separate *gharar* and ownership (*qabdh*) grounds. We rely on the narrow concession only: the structural design choice $\iota = 0$ (equivalently, $r = 0$ in credit instruments) removes the interest component by mathematical construction, not legal re-labelling [51]. Whether any particular contract built on the κ-rate is Shariah-compliant in full depends on its other contractual features and requires separate jurisprudential review.

8.2 Gharar: Is the Stopping Time Uncertain?

A potential objection is that the random stopping time τ introduces excessive uncertainty (*gharar*).

Gharar in Islamic jurisprudence refers to uncertainty about the *existence* of the object of transaction, not about its *timing* when timing is actuarially determined [22, 39].

The stopping time $\tau \sim \mathrm{Exp}(\kappa)$ is:

1. Precisely specified by its distribution, which is public and on-chain verifiable.
2. Calibrated from real economic data.
3. Analogous to the random maturity of an *ijara* (lease) that terminates when the lessee exercises an option — a widely accepted structure under AAOIFI Standard 17 [1, 4].

The stopping time introduces actuarial uncertainty (priced by κ) rather than contractual ambiguity; whether this suffices to address the *gharar* concern is a determination for Shariah scholars. Blockchain implementation (smart contracts) makes the trigger condition and the distribution parameters immutably public [41], which we submit as mitigating evidence for that review.

8.3 Maysir: Is This Speculation?

Maysir (gambling) is prohibited because it is zero-sum and divorced from productive economic activity. The κ-rate framework is designed to address the *maysir* concern because:

1. Every instrument is backed by a real asset (X_t).
2. The premium payments $\pi_t = \kappa(f_t - X_t)\,dt$ are transfers between economically motivated parties based on observed market conditions, not arbitrary bets.
3. Leverage is constrained by the Shariah hard cap (5$\times$ in the Baraka Protocol implementation [5]).

8.4 AAOIFI Considerations

- **Standard 17 (Sukuk).** The Islamic Perpetual Sukuk is designed to address the requirement that sukuk represent ownership in an asset. The stopping time redemption is structured as a purchase undertaking (*wa'd*) of the kind treated under Standard 17 [1, 4].
- **Standard 26 (Takaful).** The everlasting takaful formula is designed to address the actuarial fairness and mutual risk-sharing requirements. Pool contributions are actuarially determined, not based on investment returns.
- **Standard 21 (Financial Paper).** The iCDS, as a bilateral risk-transfer mechanism, is analogous to wa'd-based credit enhancement treated under Standard 21, provided the reference obligation is itself permissible.

Conformity with Standards 17, 21, and 26 is a determination reserved to a Shariah supervisory board.

9 Discussion

9.1 Why This Was Not Discovered Earlier

The connection between the Ackerer perpetual contract framework and the Duffie-Singleton credit risk model went unnoticed for two reasons. First, the two literatures operate in different communities (mathematical finance vs. fixed income) and are rarely cited together. Second, the Ackerer framework itself is very recent (first circulated 2023; published 2025); prior to it, there was no mathematically rigorous proof that $\iota = 0$ achieves no-arbitrage convergence in perpetual contracts, and the equivalence could not therefore be established.

The deeper reason is conceptual. Conventional finance fuses two distinct economic concepts — the timing of a payoff event and the time value of money — into a single parameter r. This fusion appeared necessary. The Ackerer framework dissolves this presumption: the stopping time distribution $\mathrm{Exp}(\kappa)$ encodes timing information probabilistically, without any predetermined interest.

9.2 The Separation Theorem

Theorem 2 (Separation of Credit Risk from Time Preference)**.** *In the Ackerer-Hugonnier-Jermann framework, the no-arbitrage price of a financial claim $\phi(X_\tau)$ at $\iota = 0$ can be written as:*

$$P_t = \underbrace{\mathbb{E}^Q[\phi(X_\tau)]}_{\text{credit risk term}} \quad \textit{without any} \quad \underbrace{\mathrm{e}^{-r(s-t)}}_{\text{time preference term}} \tag{37}$$

The timing of the payoff is embedded in the distribution of $\tau \sim \mathrm{Exp}(\kappa)$. The value of the payoff is embedded in the expectation $\mathbb{E}[\phi(X_\tau)]$. The interest parameter ι is eliminable*: setting $\iota = 0$ yields a finite, unique, no-arbitrage price with no time-preference term. (For $\iota \neq 0$ the interest parameter is not a separable discount — in the AHJ random-time representation it enters both the discount and the* stopping intensity*, $\tau \sim \mathrm{Exp}(\kappa - \iota)$. The theorem asserts that the riba-free point $\iota = 0$ is well-defined and arbitrage-free, not that interest factors out multiplicatively for all ι.)*

Proof. This is the $\iota = 0$ specialisation of Corollary 1. By Ackerer–Hugonnier–Jermann (Theorem 3), the no-arbitrage perpetual price admits the random-time representation $P_t = \mathbb{E}^{Q^a}[\mathrm{e}^{-\iota(\tau-t)}\phi(X_\tau)]$ where $\tau \sim t + \mathrm{Exp}(\kappa - \iota)$ — so the interest parameter enters *both* the discount and the stopping intensity. At $\iota = 0$ the intensity reduces to κ and the discount to unity, leaving $P_t = \mathbb{E}^{Q^a}[\phi(X_\tau)]$ with $\tau \sim \mathrm{Exp}(\kappa)$, which is finite and arbitrage-free because $\kappa > 0$ guarantees $\tau < \infty$ a.s. and the expectation exists. Interest is therefore *eliminable* — the riba-free price exists and is unique — even though, away from $\iota = 0$, it is a shift of the convergence intensity rather than a separable multiplicative discount. □

This separation is the mathematically precise statement of what Islamic finance has sought for 50 years: a proof that interest is not necessary for no-arbitrage pricing of time-dependent financial instruments.

9.3 Counterarguments and Responses

Counterargument 1: Time value of money is not riba. Some Islamic economists (notably Kahf [38] and Siddiqi [53]) argue that the time value of money can be Islamically justified through opportunity cost reasoning: capital deployed today could produce real output tomorrow, so a premium for time is fair.

Response: Even granting this argument, it does not imply that $r > 0$ is a mathematical necessity for no-arbitrage pricing. The opportunity cost of capital is already captured by κ: a high-κ instrument (fast convergence, short expected holding period) delivers value quickly and implicitly compensates for the opportunity cost of the capital committed. The Separation Theorem shows that interest is separable from, not identical to, the opportunity cost of time. The κ-rate prices the latter without encoding the former.

Counterargument 2: Conventional sukuk work; why change? Existing sukuk structures have achieved broad market acceptance. One might argue that embedding SOFR at the pricing stage is a pragmatic necessity, even if theoretically impure.

Response: Three practical problems with this position. (i) Post-LIBOR transition, sukuk issuers face SOFR repricing risk that has no Islamic instrument for hedging. (ii) Empirically, sukuk spreads are driven by credit risk not interest rates [49, 26]; the κ-rate captures spread dynamics directly. (iii) AAOIFI Standard 17 explicitly requires that sukuk represent real ownership stakes, not interest-indexed instruments — a requirement that the current market satisfies cosmetically but the κ-framework satisfies mathematically.

Counterargument 3: Stochastic κ introduces model risk. If κ must be estimated, the pricing formula inherits estimation error. Critics may argue that conventional credit models with r (observable from Treasury yields) are more tractable.

Response: This is correct, and we acknowledge it (Section 7.3). However, hazard rates λ are also estimated quantities in conventional reduced-form models [20, 9]; the estimation uncertainty of κ is no greater than that of λ. The difference is that κ can be observed from the perpetuals basis (equation 33) — a market-implied quantity — in addition to being estimated from historical default rates, giving it an additional identification channel unavailable to λ.

9.4 Limitations

Constant-parameter assumption. Closed-form results assume constant κ and σ_X. The stochastic case (Section 7.3) requires numerical methods.

Recovery structure. We assume full or floor-only recovery. Partial recovery models require adjusting ϕ accordingly.

Liquidity. The theory assumes liquid markets for both f_t and X_t. For infrastructure sukuk or agricultural takaful, the underlying may be illiquid. Liquidity adjustment to the κ-rate is an open research question.

Regulatory recognition. Formal adoption requires engagement with the AAOIFI standard-setting process, which is ongoing.

10 Simulation Validation

The theoretical results of Sections 4–7 are grounded in measurable on-chain behaviour. We validate the key claims — no riba at $\iota = 0$, protocol solvency under random stopping, and the convergence of mechanism design to the κ-rate regime — using an integrated economic simulation of the Baraka Protocol deployment on Arbitrum Sepolia.

10.1 Simulation Architecture

The simulation combines four coupled layers:

1. **cadCAD Market Engine.** A pure-Python cadCAD state machine models the continuous-time market over 720 hourly intervals per episode (30 simulated days). The five partial-state-update blocks mirror the on-chain contract logic: oracle price update, funding settlement at $\iota = 0$, trader actions, liquidations, and insurance-fund bookkeeping.
2. **Reinforcement Learning Trader.** A rule-based agent with optimal stopping-time intuition acts at each cadCAD step. Its observations are the live cadCAD state $(p_t^{\text{mark}}, p_t^{\text{index}}, F_t, \text{CF}_t, q_t, c_t, \ell_t, w_t)$; its actions ({hold, open long 1–5×, open short 1–5×, close}) inject positions directly into the cadCAD position pool.
3. **Game Theory Equilibrium Analyser.** Every 50 cadCAD steps, the simulation builds a 5×5 payoff matrix from the live price window and solves for the Nash equilibrium leverage distribution. The equilibrium is fed back as the respawn-leverage distribution for background traders, closing the game-theory feedback loop.
4. **Mechanism Design Optimiser.** At each episode boundary, a `scipy differential_evolution` pass minimises the objective

$$\mathcal{L}(\theta) = 10\,\rho(\theta) + 0.005\,\bar{\ell}(\theta) + 5\,\overline{|F|}(\theta) \tag{38}$$

   where ρ is the insolvency rate, $\bar{\ell}$ the mean liquidation count, and $\overline{|F|}$ the mean absolute funding rate. The optimal θ^* is blended (70/30) with current parameters and applied to the next episode.

10.2 Results: 5 Episodes × 720 Steps

10.3 Claim Verification

Claim 1: $\iota = 0$ implies no systematic riba. The mean net transfer from longs to the protocol was −\$660 on $\sim$ \$100,000 notional per interval, a relative imbalance of 0.66% — well within Monte Carlo noise. This is consistent with Corollary 1: at $\iota = 0$ the funding mechanism imposes no systematic wealth transfer, satisfying the jurisprudential definition of riba-freedom.

Claim 2: Protocol solvency under random stopping. Zero insolvency events were recorded across all five episodes (3,600 hourly steps, 53 liquidations). The insurance fund ended above its \$100,000 seed in every episode, finishing at \$102,835, consistent with the theoretical prediction that the takaful pool is self-sustaining when premiums are set at the actuarially fair κ-rate level.

Claim 3: Mechanism design converges to κ-regime. The optimiser converged to a tighter circuit-breaker ($F_{\max}$ reduced from 75 to 41 bps) and a higher maintenance margin (2.0% to 3.1%). Both movements are consistent with the κ-rate theory: a protocol optimising under the $\iota = 0$ design constraints should minimise $\overline{|F|}$ and maximise insurance reserves. The mechanism design layer discovered this structure endogenously (noting that its objective encodes the near-zero-funding property; see the companion simulation paper [5]).

Claim 4: Nash equilibrium leverage is sub-maximal. The mean Nash equilibrium long leverage was 2.72× and short was 3.28×, both strictly below the Shariah hard cap of 5×. This confirms that rational traders facing a $\iota = 0$ symmetric funding mechanism do not systematically exploit maximum leverage.

10.4 The Empirical κ-Rate

The empirical κ-rate is measured on real markets. On the full Cbonds universe pulls of Section 4.5, the 195-instrument cross-section gives median $\hat{\kappa} \approx 0.02$ per annum (sovereign subset 0.019) with the predicted credit ordering, and the 54-sukuk constant-maturity histories give mean-reverting long-run levels of $\approx$ 1–3% (investment grade) to 5–7% (high yield) per annum. Real credit-aggregate OAS series corroborate the high-yield end of the band: ICE BofA indices (FRED) imply $\hat{\kappa} = \text{OAS}/(1-\delta)$ of 0.104 per annum (emerging-market high-yield mean; 0.079 current) and 0.081 (US high-yield) at $\delta = 0.60$ — all identified with zero interest input. The protocol simulation above complements this with *computability*: $\hat{\kappa}$ is recoverable from on-chain-observable quantities (insurance-fund flows, funding rates, open interest), at a level ($\approx$ 0.08 per annum) inside the measured high-yield credit band, so a live $\iota = 0$ protocol can publish its own

Table 2: Per-episode simulation outcomes.

Ep.	Ins. Fund ($)	Liq.	Insolv.	RL PnL ($)	$\overline{\lvert F\rvert}$ (bps)
1	101,175	11	No	+117	70.1
2	100,782	7	No	+310	58.0
3	101,139	9	No	0	50.3
4	101,889	11	No	0	44.2
5	102,835	15	No	0	39.6
Total	—	**53**	**0/5**	**+427**	**52.4**

Table 3: Mechanism design parameter evolution.

Parameter	Initial	Final	Change
$F_{\max}$ (bps)	75.0	40.8	−45%
Maintenance margin (%)	2.0	3.1	+55%
Liquidation penalty (%)	1.0	1.3	+31%
Insurance split (%)	50.0	60.8	+22%

$\hat{\kappa}$ in real time, with the real-data estimates above as the market calibration.

11 Conclusion

We have established that the random time representation of perpetual futures contracts — a result from contemporary mathematical finance [3] — is formally isomorphic to the reduced-form credit risk pricing formula of Duffie & Singleton [19]. The isomorphism maps the convergence intensity κ to the default hazard rate λ, and the interest parameter ι to the risk-free discount rate r. Setting $\iota = r = 0$, which Ackerer et al. prove achieves unique no-arbitrage pricing for perpetual contracts, yields a complete, no-arbitrage pricing framework for credit instruments that requires no positive interest rate.

The implications for Islamic finance are substantial. The κ-rate provides a mathematically rigorous, riba-free pricing foundation — product-layer questions remain with Shariah scholars — for: (i) Islamic Perpetual Sukuk, priced at the expected asset value at a random redemption event; (ii) Everlasting Takaful contracts, with premiums given by the closed-form everlasting put formula; (iii) Islamic Credit Default Swaps, transferring credit risk without any interest component; and (iv) Sovereign Perpetual Sukuk for infrastructure finance, with κ calibrated from project timelines.

The deeper contribution is the Separation Theorem (Theorem 2): interest is a separable, eliminable component of no-arbitrage pricing, not an intrinsic mathematical necessity. The stopping time distribution carries all timing information previously attributed to the interest discount. Empirical corroboration comes from European credit markets that continued to function efficiently at $r \leq 0$ during 2014–2022 (Section 4.4).

This is the precise mathematical statement that Islamic finance has required but lacked: riba is not a logical prerequisite for fair pricing of time-dependent financial instruments. Future work should address stochastic κ estimation from panel sukuk data, liquidity premia in the κ-yield curve, and the macroeconomic properties of a monetary system anchored to κ rather than r.

"And Allah has permitted trade and forbidden riba."
(Al-Qur'an, Al-Baqarah 2:275)

CRediT Author Contribution Statement

Shehzad Ahmed: Conceptualization, Formal analysis, Methodology, Software, Visualization, Writing — original draft. **Rafiqul Bhuyan:** Conceptualization, Methodology, Supervision, Writing — review and editing.

Declaration of Competing Interest

S. Ahmed has developed an open-source smart-contract implementation of the pricing framework described in this paper (Baraka Protocol, `https://github.com/Arcus-Quant-Fund/BarakaDapp`). No financial interest or commercial funding was involved. The remaining authors declare no competing interests.

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