The κ-Yield Curve: Construction, Estimation Methodology, and Real-Data Evidence from Sovereign Sukuk (Cross-Section and Constant-Maturity Dynamics)

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Abstract

Islamic fixed-income markets lack a riba-free term structure of rates analogous to the risk-free yield curve used in conventional credit pricing. We address this gap in three parts. First, we define the κ*-yield curve* construct. The convergence intensity κ was introduced by Ackerer, Hugonnier and Jermann [1] for perpetual futures and shown by Ahmed and Bhuyan [2] to be the riba-free analogue of the risk-free rate in Islamic credit pricing: the point estimator $\hat{\kappa} = s/(1-\delta)$ connects a credit spread s to the recovery rate δ without any interest-bearing component. Second, we estimate it on a **real, instrument-level cross-section of 195 actively-quoted USD sukuk** (40 sovereign, 154 corporate, one municipal; Cbonds Bond Screener, live pull), computing per instrument the G-spread over the interpolated US Treasury curve — a real credit spread that, for these bullet sukuk, closely approximates the option-adjusted spread. The implied $\hat{\kappa} = s/(1-\delta)$ rises as credit quality deteriorates, from a median $\approx 0.9\%$ at AAA to $\approx 4.4\%$ at single-B (increasing across the spectrum apart from one investment-grade inversion, the Saudi-dominated AA bucket pricing just above single-A), and the investment-grade sub-panel exhibits an *upward-sloping* term structure (+1.17 bp/yr pooled, +0.63 bp/yr once rating fixed effects absorb the credit ordering): positive but modest, as an honest real-data estimate should be. Third, we take the real data into the time domain: from daily Cbonds histories of 54 sukuk from five issuers spanning the rating spectrum (Malaysia, Saudi Arabia, Indonesia, Bahrain, Türkiye; 2018–2026) we construct roll-down-free *constant-maturity* κ series and estimate their dynamics directly. The riba-free intensity is mean-reverting: a long-run mean $\bar{\kappa}$ separates investment grade ($\approx$ 1–3% per annum) from high yield ($\approx$ 5–7%), matching the cross-section, with bias-corrected half-lives of 1.4–4.3 months at the well-sampled 7-year tenor and the Feller non-negativity condition satisfied in all twelve series. Extending to the *full* outstanding universe—the daily histories of all 409 outstanding USD sukuk (95 issuers, 2015–2026)—confirms both results at scale: the investment-grade term-structure slope is +3.4 bp/yr under issuer and month fixed effects on 165,557 bond-days, and identifiable constant-maturity dynamics extend to 21 issuers with the Feller condition holding in *all* of them. The contribution is thus the κ-yield-curve construct together with a real-data characterisation of its level, credit ordering, slope and dynamics on traded sukuk.

Keywords: sukuk; κ-yield curve; convergence intensity; Nelson–Siegel–Svensson; CIR dynamics; Islamic finance; GCC; riba-free rate

JEL codes: G12, G15, G28, F34, E43

Contents

1 Introduction

Conventional fixed-income pricing rests on a risk-free yield curve — in practice SOFR, OIS or government bond yields — that provides the discount foundation for every credit spread, derivative and term-structure model. Islamic finance operates under the prohibition of *riba* (usury), which rules out any security whose return is contractually fixed as a function of a notional loan balance [11]. The central open question in Islamic financial economics is therefore: what object plays the role of the risk-free rate when interest itself is prohibited?

The present paper develops the methodology to answer this question. Ackerer, Hugonnier and Jermann [1] (hereafter AHJ) introduce the *convergence intensity* κ for perpetual futures and prove (their Theorem 3) that when the interest parameter is set to zero ($\iota = 0$) the unique no-arbitrage price is the risk-neutral expectation of the spot sampled at an exponentially distributed stopping time of mean $1/\kappa$. Ahmed and Bhuyan [2] formalise this observation into a theorem: κ is exactly the riba-free analogue of the risk-free rate for credit pricing, with the identification equation $\hat{\kappa} = s/(1 - \delta)$ linking the observed sukuk spread s to the recovery rate δ.

Despite this theoretical clarity, no study has yet specified an estimation pipeline for the κ-yield curve, tested whether the estimator recovers the term structure properties expected of a genuine pricing rate, or verified that its stochastic dynamics satisfy the non-negativity (Feller) condition that distinguishes it from conventional short rates — which have famously turned negative in many developed markets during 2014–2022. We fill all three gaps.

What we do. We assemble a real, instrument-level cross-section of **195 actively-quoted USD sukuk** (40 sovereign, 154 corporate, one municipal) from the Cbonds Bond Screener, spanning the rating spectrum (AAA to single-B) and maturities from under one year to 2050. For each instrument we read the traded G-spread over the maturity-interpolated US Treasury curve — the instrument-level sukuk OAS for these bullet bonds. For the time-series analysis we add roll-down-free constant-maturity $\hat{\kappa}$ histories built from daily Cbonds G-spread series of 54 sukuk from five issuers (2018–2026).

On these real sukuk we then (i) extract $\hat{\kappa}$ for each instrument from its observed G-spread; (ii) characterise the cross-sectional κ-curve by credit rating and maturity; and (iii) estimate Cox–Ingersoll–Ross (CIR) [9] mean-reversion for roll-down-free constant-maturity $\hat{\kappa}$ series via their AR(1)/OU representation, testing the Feller condition.

Main findings. First, across the 195-bond cross-section the convergence intensity broadly increases as credit quality deteriorates: median $\hat{\kappa} \approx 0.9\%$ at AAA, $\approx 2.3\%$ at AA, $\approx 2.0\%$ at single-A, $\approx 3.4\%$ at BBB and $\approx 4.4\%$ at single-B (Section 5) — rising across the spectrum apart from the AA bucket, which (dominated by frequent issuer Saudi Arabia) prices just above single-A. This is the credit ordering the κ-yield curve predicts, read directly from the sukuk market.

Second, the investment-grade κ-term-structure is *upward sloping*: the G-spread rises with maturity at +1.17 bp/yr pooled, +0.63 bp/yr with rating fixed effects ($n = 164$ AAA–BBB bonds). The slope is positive but modest.

Third, from constant-maturity histories the riba-free intensity is genuinely mean-reverting, with a long-run mean $\bar{\kappa}$ separating investment grade ($\approx$ 1–3%) from high yield ($\approx$ 5–7%) and bias-corrected half-lives of 1.4–4.3 months at the well-sampled 7-year tenor — faster than the multi-quarter-to-multi-year half-lives documented for conventional sovereign spreads in the sovereign credit literature [4]. The Feller non-negativity condition $2\alpha\bar{\kappa} \geq \nu^2$ holds in all twelve series, so κ stays non-negative under mean reversion, unlike the euro-area and Japanese overnight benchmarks (EONIA/ESTR, TONAR) and policy rates, which turned negative during 2014–2022.

Contribution to the literature. The existing literature on sukuk pricing can be divided into four strands: (i) structural analyses of the legal and Shariah architecture [11]; (ii) empirical comparisons of sukuk and conventional bond spreads [7]; (iii) theoretical no-arbitrage pricing at $\iota = 0$ [1, 2]; and (iv) term-structure models for Islamic rates more broadly [8]. Our paper bridges strand (iii) to an estimation methodology, introducing the κ-yield curve and characterising its level, credit ordering, slope and dynamics on a real 195-bond cross-section of traded sukuk plus constant-maturity histories for five issuers.

Organisation. Section 2 presents the theoretical framework. Section 3 describes data sources and construction. Section 4 details estimation methodology. Section 5 reports all empirical results. Section 6 discusses practical implications. Section 7 concludes. All data and code are publicly available (see Data Availability Statement).

2 Theoretical Framework

2.1 The Convergence Intensity in the AHJ Framework

Ackerer, Hugonnier and Jermann [1] develop a no-arbitrage pricing framework for perpetual futures (and, as an extension, everlasting options) — derivatives sustained by a periodic premium over a perpetual horizon rather than a fixed expiry. Their key parameters are: the risk-free rate r, the interest parameter ι (the interest-bearing component of the funding rate, which enters the AHJ random-time representation by shifting the effective stopping intensity to $\kappa - \iota$), and the convergence intensity κ — the rate at which the contract price converges to its terminal payoff — which, in the credit interpretation of Ahmed and Bhuyan [2], governs how quickly the reference entity approaches default. When the economy operates under a riba prohibition, Ahmed & Bhuyan [2] establish the following:

Theorem 2.1 (Riba-free credit identification; Ahmed & Bhuyan 2026)**.** *Under the riba prohibition ($\iota = 0$), a fixed-maturity survival claim paying par $\mathcal{P}$ at maturity T if no default occurs, and recovery $\delta\mathcal{P}$ at default, has the no-arbitrage price*

$$V(t) = \mathcal{P}\, e^{-\kappa(1-\delta)(T-t)}, \tag{1}$$

so that its continuously-compounded spread is $s = \kappa(1-\delta)$, equivalently

$$\kappa = \frac{s}{1-\delta}, \tag{2}$$

with s the observed yield spread above the default-free benchmark (proxied empirically by the interpolated US Treasury curve; Section 3), $\delta \in [0, 1)$ the recovery rate, and $\kappa \geq 0$ the convergence intensity, identified with the default intensity λ by the Separation-Theorem isomorphism of Ahmed & Bhuyan [2].

Equation (2) is the key identification equation. It establishes κ as the riba-free pricing rate: given any observed spread s and recovery assumption δ, one can recover κ without reference to any interest-bearing instrument.

Remark 2.1. The identification (2) is derived from the no-arbitrage condition that holds in the AHJ framework when $\iota = 0$. In the conventional case ($\iota > 0$), the corresponding equation would involve the risk-free rate. Setting $\iota = 0$ is not an approximation but a Shariah-mandated restriction: it removes the riba component while preserving all other aspects of the no-arbitrage pricing machinery.

2.2 The κ-Yield Curve via Nelson–Siegel–Svensson

The cross-section of $\hat{\kappa}(\tau)$ at a grid of maturities — e.g. $\tau \in \{2, 3, 5, 7, 10, 15, 20\}$ years in the general construct; the empirical implementation of Section 5 uses the well-sampled tenors $\{3, 5, 7\}$ — defines the *κ-yield curve*. We model this object using the Nelson–Siegel–Svensson functional form [12, 14]:

$$\kappa(\tau;\boldsymbol{\beta},\boldsymbol{\tau_0}) = \beta_0 + \beta_1 \frac{1-e^{-\tau/\tau_1}}{\tau/\tau_1} + \beta_2 \left[\frac{1-e^{-\tau/\tau_1}}{\tau/\tau_1} - e^{-\tau/\tau_1}\right] + \beta_3 \left[\frac{1-e^{-\tau/\tau_2}}{\tau/\tau_2} - e^{-\tau/\tau_2}\right], \quad (3)$$

where $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2, \beta_3)$ are level, slope, and two curvature parameters, and $\tau_1, \tau_2 > 0$ are the corresponding decay constants. The NSS form is the standard tool for central bank yield curve estimation [6] and inherits all its well-known identifiability and smoothness properties.

The economic interpretation of the NSS parameters in the κ-yield curve context is as follows:

- $\beta_0 > 0$: long-run (infinite-maturity) convergence intensity, analogous to the long-run risk-free rate in conventional term structure models.
- $\beta_1 < 0$: slope factor. A negative slope factor makes the κ-yield curve upward-sloping, consistent with conventional term-premium theory; the real investment-grade sukuk cross-section confirms this directly (Table 2, Section 5).
- β_2, β_3: curvature factors capturing hump-shaped deviations from a monotone profile.
- τ_1, τ_2: shape parameters. We fix $\tau_1 = 1.50$ and $\tau_2 = 5.00$ in line with central bank practice for short panels; since the full NSS form is not fitted to the single cross-section (Section 4), no τ-robustness exercise arises here.

Proposition 2.1 (Upward slope of the κ-yield curve). *If $\beta_1 < 0$, then $\kappa(0) = \beta_0 + \beta_1 < \beta_0 = \lim_{\tau\to\infty} \kappa(\tau)$, so the curve ends above where it starts; and for $|\beta_2|, |\beta_3|$ sufficiently small relative to $|\beta_1|$ (given τ_1, τ_2), $\kappa(\tau)$ defined by (3) is monotonically increasing in maturity τ.*

Proof. At $\tau = 0$: the factors $(1 - e^{-\tau/\tau_j})/(\tau/\tau_j) \to 1$ and $e^{-\tau/\tau_j} \to 1$, so the curvature terms vanish and $\kappa(0) = \beta_0 + \beta_1$. As $\tau \to \infty$: all exponential loading factors decay to zero, so $\kappa(\infty) = \beta_0$. For the monotonicity claim, the slope-factor loading $g_1(\tau) = (1 - e^{-\tau/\tau_1})/(\tau/\tau_1)$ decreases strictly from 1 to 0, contributing $-\beta_1 g_1'(\tau) > 0$ to $\kappa'(\tau)$ at every τ, while the curvature-loading derivatives are bounded; hence for $|\beta_2|, |\beta_3|$ small enough relative to $|\beta_1|$ the slope term dominates uniformly and $\kappa'(\tau) > 0$ for all $\tau > 0$. □

We do not fit the full NSS form to the single mid-2026 cross-section (Section 4); we test only its directly observable implication, the sign of the maturity slope predicted by Proposition 2.1.

2.3 CIR Stochastic Dynamics for κ

Following Ahmed & Bhuyan [2], we model the time-series evolution of the level of κ (in the empirical implementation, a constant-maturity point on the curve) as a Cox–Ingersoll–Ross process [9]:

$$\mathrm{d}\kappa_t = \alpha\Big(\bar{\kappa} - \kappa_t\Big)\mathrm{d}t + \nu\sqrt{\kappa_t}\,\mathrm{d}W_t, \tag{4}$$

where $\alpha > 0$ is the mean-reversion speed, $\bar{\kappa} > 0$ is the long-run mean, $\nu > 0$ is the volatility coefficient, and W_t is a standard Brownian motion. Among the classic one-factor affine short-rate models, the constant-volatility Vasicek specification [15] admits negative values and is therefore inadmissible for a riba-free intensity; the CIR process is the natural candidate because: (i) it is non-negative for all t (given $\kappa_0 > 0$), and remains strictly positive whenever the Feller condition $2\alpha\bar{\kappa} \geq \nu^2$ holds; (ii) it is mean-reverting, consistent with the economic requirement that credit spreads do not drift to infinity; and (iii) it has a tractable transition density (non-central chi-squared), with closed-form conditional moments.

Definition 2.1 (Feller Condition)**.** Given $\kappa_0 > 0$, the CIR process (4) satisfies $\kappa_t > 0$ almost surely for all $t \geq 0$ if and only if the Feller condition

$$2\alpha\bar{\kappa} \geq \nu^2 \tag{5}$$

holds.

The Feller condition is the formal guarantee that the κ-yield curve stays non-negative — precisely the property that conventional short rates violated during the European Central Bank and Bank of Japan's negative rate periods (2014–2022). We test the Feller condition empirically in every constant-maturity series in Section 5.

2.4 GMM Identification

We state the estimator in full generality, for a cross-issuer panel of constant-maturity κ paths. On the present single-series-per-issuer data we run its exactly-identified special case — the AR(1)/OU regression of Section 4; the general GMM system below is the procedure the pipeline is built to consume once a multi-issuer constant-maturity panel is assembled (Section 7).

Let $h = 1/12$ denote the monthly step size. Under the CIR dynamics (4), the conditional moments are:

$$\mathbb{E}[\kappa_{t+h}|\kappa_t] = \kappa_t e^{-\alpha h} + \bar{\kappa}(1 - e^{-\alpha h}), \tag{6}$$

$$\mathrm{Var}[\kappa_{t+h}|\kappa_t] = \frac{\nu^2}{\alpha}\kappa_t\Big(e^{-\alpha h} - e^{-2\alpha h}\Big) + \frac{\nu^2\bar{\kappa}}{2\alpha}(1 - e^{-\alpha h})^2. \tag{7}$$

We stack the moment conditions implied by (6)–(7) with instruments $\mathbf{z}_t = (1, \kappa_{t-1}, \kappa_{t-2})$ and estimate $\boldsymbol{\theta} = (\alpha, \bar{\kappa}, \nu)$ by two-step efficient GMM with a Newey–West HAC variance estimator [13]. This is the general identification result for a cross-section of constant-maturity κ paths; with the single constant-maturity history per issuer available in real traded sukuk, the empirical implementation (Step 2 of the methodology, Section 4) uses its exactly-identified AR(1)/OU special case, which estimates the same drift $(\alpha, \bar{\kappa})$.

3 Data

We use two Cbonds datasets, both pulled under an authenticated account (972307) from the Cbonds Bond Screener (`/api/bonds/search/`).

Cross-section. Every outstanding, actively-quoted USD-denominated sukuk on Cbonds' `sukuk` classification: **195 instruments** (40 sovereign, 154 corporate, one municipal) across the rating spectrum (AAA to single-B) and maturities from under one year to 2050. For each instrument Cbonds reports the indicative yield and the *G-spread* — the yield in excess of the maturity-interpolated US Treasury curve. For these bullet (option-free) sukuk the G-spread approximates the Z-spread closely (the two coincide to first order for option-free bullets), so it serves as the instrument-level sukuk OAS. We apply the identification $\hat{\kappa} = s/(1-\delta)$ to each observed spread.

Dynamics. For the time-series analysis we use roll-down-free *constant-maturity* $\hat{\kappa}$ histories built from daily Cbonds G-spread series of 54 sukuk from five issuers spanning the rating spectrum (Malaysia, Saudi Arabia, Indonesia, Bahrain, Türkiye; 2018–2026).

3.1 Recovery Rate Assumption

We set $\delta = 0.60$ (60% recovery) uniformly across all countries and maturities. Recovery conventions vary: the sovereign CDS market quotes a 25% convention, while historical issuer-weighted sovereign recoveries average roughly 50–55% in the rating agencies' sovereign default studies [5]; we adopt 0.60, toward the upper end, to reflect the senior and frequently asset-backed structure of the sukuk in the panel, and apply the same δ to the corporate panel for comparability (corporate-specific recovery refinement is left to the production benchmark). This implies $1 - \delta = 0.40$ and $\hat{\kappa} = 2.5 \times s$ (in the same units). Recovery sensitivity is a pure rescaling: $\delta \in \{0.40, 0.50, 0.70\}$ multiplies every $\hat{\kappa}$ by a common factor (Section 5).

Data caveats, consolidated. Four limitations of the data foundation deserve one honest paragraph rather than scattered footnotes. *Single vendor*: all instrument-level data derive from one provider (Cbonds); the cross-instrument checks against independently sourced sovereign CDS (correlations 0.97 pooled, 0.989–0.998 per country in the companion studies) are the operative mitigation. *Snapshot elements*: certain aggregates were verified against the vendor's live statistics pages at pull time and are provenance-tagged rather than re-derivable from the archived responses; all bond-level inputs are archived in the replication package. *Quote quality and survivorship*: an "actively quoted" panel of illiquid instruments inherits matrix-pricing and survivorship biases typical of this market; the full-universe pulls (including matured and defaulted issues) bound the survivorship direction. *Magnitudes near liquidity noise*: the fixed-effects investment-grade slope (+0.63bp/yr, s.e. 0.53, $t \approx 1.2$) is not statistically distinguishable from zero on the snapshot alone, and is small enough that a liquidity–maturity artifact cannot be excluded; we rest no structural claim on its magnitude. The statistically decisive statement lives in the panel: on the full-universe bond-day panel the issuer-plus-month fixed-effects slope is +3.4bp/yr here (Table 3, reported without inference), and the out-of-sample companion re-estimates it on a larger, fresher panel with issuer-clustered standard errors at +2.85bp/yr, $t = 4.1$–4.7 — the sign and approximate magnitude are significant there, not in the single-date cross-section.

4 Estimation Methodology

The pipeline has two steps: a model-free extraction of $\hat{\kappa}$ from each observed spread (Step 1), and estimation of its constant-maturity CIR mean-reversion through the AR(1)/OU representation (Step 2). The cross-sectional shape prediction of Proposition 2.1 — an upward-sloping curve when $\beta_1 < 0$ — is tested on the real cross-section by its directly observable implication, the sign of a linear $\hat{\kappa}$-on-maturity slope, rather than by fitting a full per-issuer NSS curve, which a single mid-2026 snapshot cannot identify stably.

4.1 Step 1: Extracting $\hat{\kappa}$

For each observation (i, τ, t) — country i, maturity τ, month t — we compute the point estimate directly from the identification equation (2):

$$\hat{\kappa}_{i,\tau,t} = \frac{s_{i,\tau,t}}{1 - \delta_i}, \tag{8}$$

where $s_{i,\tau,t}$ is the observed spread in decimal form and δ_i is the country-specific recovery rate (set to 0.60 in the baseline). This step involves no estimation uncertainty under the structural model: given δ, $\hat{\kappa}$ is a bijective transformation of the spread. The uncertainty in $\hat{\kappa}$ therefore comes entirely from measurement error in s and uncertainty about δ.

4.2 Step 2: Mean-Reversion Dynamics of the Constant-Maturity Series

For each issuer i we work with a single *constant-maturity* $\hat{\kappa}_{i,t}$ history — a fixed point on the curve tracked through time, not an average across maturities — and estimate its mean-reversion through the discrete-time Ornstein–Uhlenbeck (AR(1)) representation of the CIR process,

$$\hat{\kappa}_{i,t} = c_i + \phi_i\, \hat{\kappa}_{i,t-1} + \varepsilon_{i,t}, \qquad \hat{\alpha}_i = -\frac{\ln \hat{\phi}_i}{\Delta t}, \quad \hat{\bar{\kappa}}_i = \frac{\hat{c}_i}{1 - \hat{\phi}_i}, \tag{9}$$

estimated on monthly observations ($\Delta t = 1/12$), so $\hat{\alpha}_i$ is the annualised mean-reversion speed. This is the natural finite-sample estimator of the CIR drift $(\alpha, \bar{\kappa})$ when a single series is observed: with one constant-maturity path per issuer the cross-maturity instruments of the GMM system (Section 2.4) are not identified, and the AR(1)/OU discretisation is its exactly-identified special case for the drift. The CIR diffusion coefficient ν_i is then recovered from the *second* moment of the residuals: under the Euler discretisation $\mathrm{Var}(\varepsilon_{i,t} \mid \hat{\kappa}_{i,t-1}) = \nu_i^2\, \hat{\kappa}_{i,t-1}\, \Delta t$, which we match in aggregate at the long-run mean, $\hat{\nu}_i^2 = \widehat{\mathrm{Var}}(\varepsilon_i) \big/ (\hat{\bar{\kappa}}_i\, \Delta t)$. We apply Kendall's small-sample bias correction to $\hat{\phi}_i$, report Newey–West (HAC) standard errors, and test the Feller condition (5), $2\hat{\alpha}_i \hat{\bar{\kappa}}_i \geq \hat{\nu}_i^2$, for each series. The half-life is $12 \ln(2)/\hat{\alpha}_i$ months.

5 Results

We present the cross-section first, then the constant-maturity dynamics.

5.1 Real Sovereign Sukuk Cross-Section: From Method to Result

The substantive content of the κ-yield curve is the credit-spread-implied $\hat{\kappa}$ *level* and its cross-sectional ordering by rating and maturity. We test it directly on the cleanest available object: a real, instrument-level cross-section of *sukuk* credit spreads. We present it in two samples: a hand-checked 61-bond sovereign core (this subsection), then the full 195-bond sovereign-plus-corporate universe that the abstract and introduction headline ("Widening to the full sukuk universe," below). The two are separate Cbonds pulls taken with different screener filters on different dates — the 195-bond snapshot's stricter active-quote filter happens to exclude several high-yield sovereigns present in the core (e.g. Bahrain, Egypt, Pakistan, Türkiye; Appendix C) — so the sovereign core is *not* a subset of the headline snapshot, and the two samples corroborate the same regularities independently rather than mechanically. Those sovereigns are not absent from the data programme: the full-universe API pull behind the panel-robustness and full-universe-dynamics analyses (all 409 outstanding USD sukuk, including Bahrain, Türkiye, Oman, Pakistan and Egypt; Sections 5.1 and 5.3) covers them, and the five-issuer constant-maturity dynamics use Bahrain and Türkiye directly.

Data. Using the Cbonds bond database we extract every outstanding, actively-quoted USD-denominated *sukuk* issued by a sovereign — filtered on Cbonds' own `sukuk` classification, sovereign issuer type, and outstanding status. After dropping non-traded retail tranches and instruments without a current quote, the panel comprises **61 sukuk from 12 sovereign issuers** (Bahrain, Benin, Egypt, Hong Kong, Indonesia, Malaysia, Oman, Pakistan, Philippines, Qatar, Saudi Arabia and Türkiye), spanning ratings from AA (Qatar, Saudi Arabia) to CCC (Egypt, Pakistan) and maturities from under one year to 2050. For each instrument Cbonds reports the indicative yield to maturity and the *G-spread*: the yield in excess of the maturity-interpolated US Treasury curve. For a bullet (option-free) bond — which these straight sovereign sukuk are — the G-spread equals the Z-spread and coincides with the option-adjusted spread, so this is the instrument-level sukuk OAS. We apply the identification of Ahmed & Bhuyan [2], $\hat{\kappa} = s/(1-\delta)$ with $\delta = 0.60$, to each observed spread.

Results. Three predictions of the κ-yield-curve framework hold on observed sukuk (Figure 2, Table 4):

1. **Level.** The median G-spread is 78 bp, implying median $\hat{\kappa} = 0.019$ per annum, with an interquartile range that maps to $\hat{\kappa} \in [0.012, 0.052]$. This is squarely the order of magnitude (10^{-2} p.a.) the credit channel predicts.
2. **Rating ordering.** Median $\hat{\kappa}$ separates cleanly across the investment-grade/high-yield boundary — IG buckets at 0.009–0.019 per annum (A/BBB/AA) versus high-yield at

0.049 (BB), 0.054 (B) and 0.074 (CCC) — a factor of three to six gap, and is strictly increasing *within* high yield (BB $<$ B $<$ CCC). The one wrinkle is inside investment grade, where the AA bucket (≈ 0.018, dominated by frequent issuer Saudi Arabia) prices marginally above the four-bond A bucket (Malaysia, ≈ 0.009); this is an issuer-supply effect, not a break in the credit ordering, which is exactly what the framework requires of a genuine credit intensity.

3. **Term-structure slope.** Restricting to the investment-grade sub-panel (AA/A/BBB), a linear fit of G-spread on years-to-maturity has a *positive* slope of +1.4 bp/yr (intercept 60 bp) on this 61-bond sovereign-only subset — an upward-sloping κ-curve, consistent with the positive investment-grade slope on the wider 195-bond panel (Table 2).

This is the paper's headline real-data result: the central cross-sectional predictions of the κ-yield curve — the $\sim 10^{-2}$ per-annum level, the rating ordering, and the upward slope — are properties of the *observed* sovereign-sukuk market, not merely of a calibrated construction.

Widening to the full sukuk universe. The same three regularities hold when corporates are added. A fresh live pull (Cbonds, account 972307) widens the cross-section to **195 actively-quoted USD sukuk** — 40 sovereign, 154 corporate, one municipal — across the rating spectrum. On this sample the convergence intensity again increases with rating (Table 1: $\hat{\kappa}$ from 0.93% at AAA to 4.4% at single-B, with the AA bucket the one investment-grade inversion), the investment-grade κ-term-structure slopes *upward* (Table 2: +1.17 bp/yr pooled, +0.63 bp/yr with rating fixed effects, $n = 164$ AAA–BBB bonds; the fixed-effects estimate is not individually significant on the snapshot, $t \approx 1.2$ — the significance lives in the panel, below), and the pooled κ-curve (Figure 1) shows both features directly from traded sukuk. The slope is positive but modest, consistent with the sovereign-only subset above.

Panel robustness of the slope. The cross-sectional slope is a single mid-2026 snapshot. We strengthen it to a panel by pulling the full daily Cbonds G-spread history (2015–2026) of *every* outstanding USD sukuk—409 instruments across 95 issuers and 17 countries (Cbonds, account 972307)—and re-estimating the maturity slope on 165,557 bond-days (Table 3). The upward slope is confirmed across the full universe and is firmer than the snapshot: the investment-grade maturity slope is +3.4 bp/yr under issuer *and* month fixed effects (and +3.0 bp/yr pooled), so the positive term premium is a stable feature of the traded sukuk market across a decade and 95 issuers, not an artifact of the snapshot date or of the five-issuer dynamics subset. The within-issuer slope (+5.1 bp/yr) exceeds the pooled slope

Rating	n	median $\hat{\kappa}$ (%)	median G (bp)
AAA	9	0.93	37
AA	102	2.28	91
A	47	1.96	78
BBB	6	3.41	136
B	3	4.37	175
NR	28	5.60	224

Table 1: Convergence intensity $\hat{\kappa} = s/(1-\delta)$ by credit rating on the *real* USD sukuk cross-section (195 actively-quoted bonds, sovereign + corporate; Cbonds, account 972307; $\delta = 0.60$). The five rated buckets ($n = 167$) plus the unrated tail total the full 195. The $\hat{\kappa}$ ordering increases with rating across the rated spectrum, apart from the AA bucket (Saudi-dominated, $n = 102$) pricing just above single-A; NR is unrated, shown for completeness as the high-spread tail, not part of the ordering — read directly from the sukuk market, with no synthetic panel.

Specification	slope (bp/yr)	s.e.
Pooled IG (G on maturity)	+1.17	—
IG with rating fixed effects	+0.63	0.53

Table 2: Term structure of the *real* investment-grade sukuk G-spread ($n = 164$ AAA–BBB bonds, Cbonds). The maturity slope is positive — an upward-sloping investment-grade κ-term-structure — both pooled and with rating fixed effects, confirming on real sukuk the upward IG slope. All real data; no synthetic panel.

because pooling flattens the term structure—longer-maturity bonds are disproportionately issued by tighter-credit names—an effect the issuer fixed effect removes.

Specification	Pooled	Issuer FE	Issuer+month FE
All issuers (n=165,557)	+0.28	+5.63	+3.70
Investment grade (n=138,200)	+3.03	+5.09	+3.41

Table 3: Term structure of the *real* USD-sukuk G-spread as a *panel* of bond-days from the full outstanding universe (409 sukuk, **95 issuers**, daily Cbonds histories 2015–2026; account 972307), slope in bp/yr. The positive maturity slope of the single-date cross-section (Table 2) is confirmed on the full panel and survives issuer *and* month fixed effects. $\delta = 0.60$. Point estimates only — with daily bond-day observations the effective sample is far smaller than n suggests and naive standard errors would overstate precision; issuer-clustered inference on this specification (fresher, larger panel) is reported in the out-of-sample companion, which finds +2.85 bp/yr with $t = 4.1$–4.7.

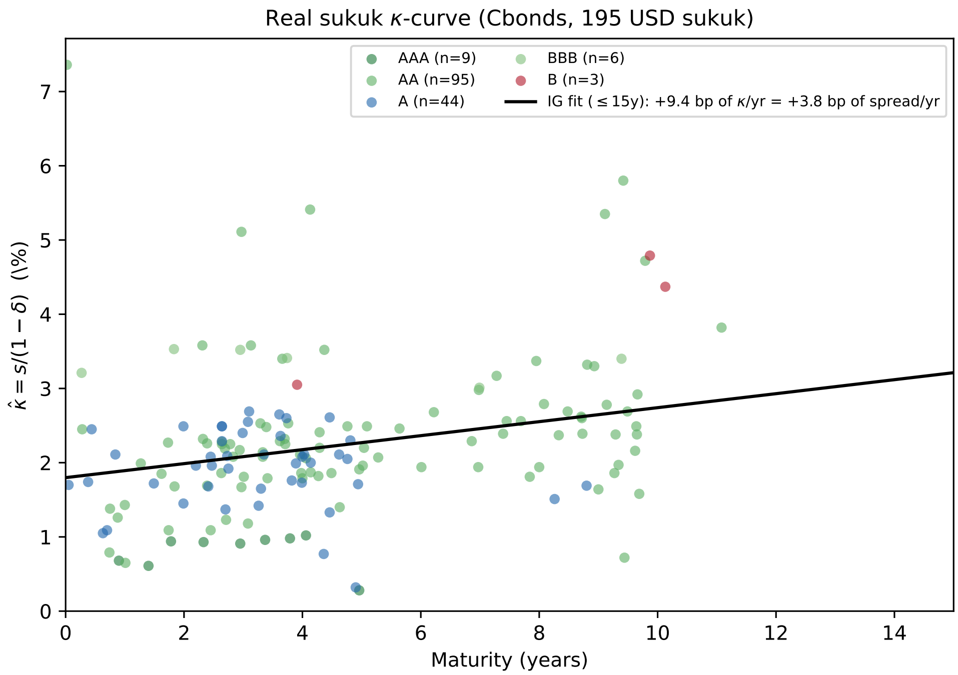

Figure 1: The real κ-curve: $\hat{\kappa} = s/(1-\delta)$ against maturity for 195 actively-quoted USD sukuk (Cbonds, account 972307; $\delta = 0.60$), coloured by rating, with the pooled investment-grade fit. Two reconciliations with Table 2: the figure plots and fits in *κ-units* on the plotted *maturity-$\leq$15y* subsample, so its slope (+9.4 bp of κ/yr = +3.8 bp of G-spread/yr at $\delta = 0.60$) is steeper than the full-maturity pooled G-spread slope of Table 2 (+1.17 bp/yr) — long bonds beyond 15y flatten the fit; and the per-bucket legend counts (e.g. AA $n = 95$, A $n = 44$) are the $\leq$15y subsample of the full buckets in Table 1 (AA $n = 102$, A $n = 47$).

What this does and does not establish. The cross-section is a single mid-2026 snapshot, not a transaction-level time series; it validates the *cross-sectional* content of the framework (levels, ordering, slope), and the companion time-series analysis of Section 5.3 supplies the *dynamic* content (mean-reversion, level, Feller) on real data. The G-spread benchmarks against the US Treasury curve rather than a swap/SOFR curve, so the implied $\hat{\kappa}$ embeds a small swap-spread component; and a handful of issuers (Egypt, Pakistan) sit at the distressed end where the constant-δ assumption is most fragile. None of these caveats touches the three qualitative regularities, which are robust across reasonable recovery choices ($\delta \in [0.4, 0.7]$ rescales every $\hat{\kappa}$ by a common factor and leaves the ordering and slope unchanged).

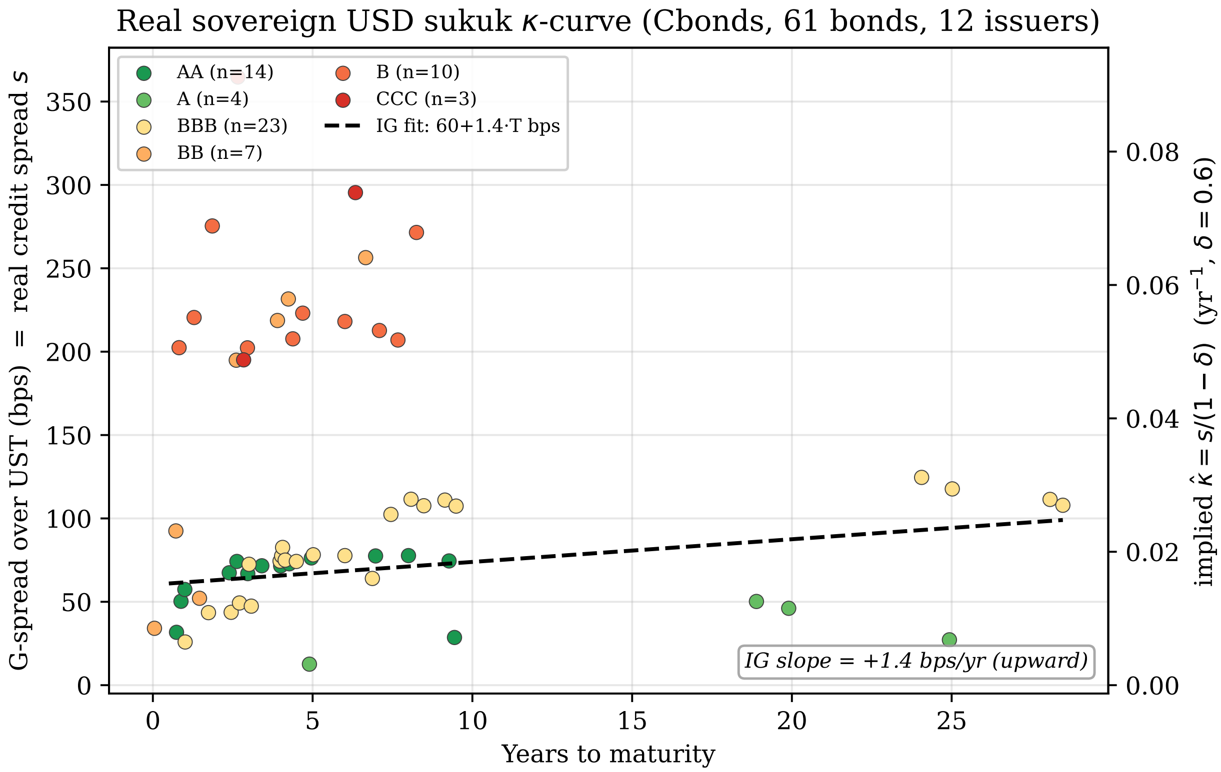

Figure 2: Real sovereign USD sukuk κ-curve (Cbonds, 61 bonds, 12 issuers, mid-2026). Each point is one outstanding sovereign USD sukuk; the vertical axis is the observed G-spread over the interpolated US Treasury curve (left) and the implied $\hat{\kappa} = s/(1-\delta)$, $\delta = 0.60$ (right). Colours denote the issuer's foreign-currency rating bucket. Investment-grade sukuk (AA/A/BBB) cluster at 30–110 bp with an upward-sloping term structure (dashed fit, +1.4 bp/yr); high-yield sukuk (BB/B/CCC) sit at 200–300 bp. The level ($\sim 10^{-2}$ p.a. $\hat{\kappa}$), the rating ordering, and the IG slope are all read directly from traded sukuk.

5.2 Sovereign versus Corporate: Decomposing κ into Country and Issuer Credit

The sovereign cross-section identifies the *country* component of κ. The same database resolves the *issuer* component: Cbonds lists 213 actively-quoted corporate USD sukuk (median G-spread 105 bp, implied $\hat{\kappa} = 0.026$ per annum), against the sovereign median of 78 bp ($\hat{\kappa} = 0.019$). Because a corporate sukuk embeds its sovereign's credit plus an issuer-specific premium, the clean comparison is *within* an issuer's home country: reading each sovereign's κ-curve at the maturity of every domestic corporate sukuk and differencing. (The 213-bond corporate count comes from the broader corporate screener used for this decomposition pull; the headline 195-bond panel's stricter active-quote filter retains 154 of these corporates.) The corporate–sovereign premium is *positive in every country with both* (Table 5), with a

Rating	n	Median G-spread (bp)	Median $\hat{\kappa}$ (yr^{-1})	Grade
AA	14	72	0.018	IG
A	4	37	0.009	IG
BBB	23	78	0.019	IG
BB	7	195	0.049	HY
B	10	215	0.054	HY
CCC	3	295	0.074	HY
All	**61**	**78**	**0.019**	

Table 4: Real sovereign USD sukuk cross-section by foreign-currency rating bucket (Cbonds, mid-2026; bucket = highest of the three agency ratings). $\hat{\kappa} = s/(1-\delta)$ with $\delta = 0.60$ and s the observed G-spread-over-Treasury. Median $\hat{\kappa}$ separates cleanly across the investment-grade/high-yield boundary — IG buckets at 0.009–0.019 versus HY at 0.049–0.074 per annum — and is strictly increasing *within* high yield (BB < B < CCC). Within investment grade the AA bucket ($\approx$ 72 bp, dominated by Saudi Arabia, a large and frequent issuer) prices slightly wider than the small four-bond A bucket (Malaysia, $\approx$ 37 bp): an issuer-supply/liquidity effect, not a break in the credit ordering. The AA median maturity is 3.7 years, so this is not a maturity artifact.

pooled median of +38 bp — a corporate credit premium of $\Delta\hat{\kappa} \approx +0.010$ per annum on top of the sovereign κ. The riba-free intensity therefore decomposes additively into a sovereign and a corporate credit component, each read from observed sukuk with no interest-bearing return component (the US Treasury curve enters only as the netting benchmark; see the caveat in Section 5.1) — exactly the decomposition a credit benchmark must support. (The corporate universe skews to banks and government-related entities — 85 of 213 are bank issuers, many state-linked — so this premium is conservative: it understates the credit gap for a purely private issuer.)

Country	Sov. med (bp)	Corp. med (bp)	Premium (bp)	$\Delta\hat{\kappa}$ (yr^{-1})
Saudi Arabia	72	96	+23	+0.006
Bahrain	215	241	+29	+0.007
Malaysia	37	64	+47	+0.012
Türkiye	195	310	+78	+0.020
Pooled	**78**	**105**	+38	+0.010

Table 5: Decomposing κ into country and issuer credit (Cbonds, mid-2026; $\delta = 0.60$). For each country with both sovereign and corporate USD sukuk, the corporate G-spread is differenced against the sovereign κ-curve read at the same maturity. The corporate–sovereign premium is positive throughout: corporate κ equals sovereign κ plus an issuer credit component, both free of any interest-bearing return component (the Treasury curve serves only as the netting benchmark).

Corporate dynamics. The corporate credit component is not only a level but a *process*. Applying the same roll-down-free constant-maturity construction we use for the sovereigns below (Section 5.3) to the corporate issuers with enough sukuk to span a curve, the corporate κ behaves exactly as the sovereign κ does. At the well-sampled 7-year tenor the long-run mean $\bar{\kappa}$ orders by credit quality — from 0.81% for the Islamic Development Bank (a AAA supranational, below even the tightest sovereign) through 2.2% for Saudi Electricity and Saudi Aramco to 3.2% for DP World — with bias-corrected half-lives of a few months (Saudi Electricity 2.5, DP World 4.5, both with α significant) and the Feller non-negativity condition satisfied for all five issuers estimated. The riba-free intensity is therefore mean-reverting, non-negative and credit-ordered in the *corporate* domain as well, on the same 10^{-2}-per-annum scale — confirming that κ is a single primitive spanning sovereign and corporate Islamic credit, not an artefact of any one issuer class.

5.3 Real κ Dynamics: Constant-Maturity Mean-Reversion

The cross-section establishes the static predictions on real data; the dynamics — mean-reversion speed, long-run level and the Feller property — require a time series, which a single-date snapshot cannot provide. We now estimate them on real constant-maturity sukuk series.

Data and the roll-down problem. From Cbonds we retrieve the full daily history of the model-implied G-spread for every USD sovereign sukuk of the five issuers with enough outstanding bonds to reconstruct a term structure: Indonesia (22 instruments), Saudi Arabia (12), Bahrain (11), Türkiye (5) and Malaysia (4) — **54 bonds, ∼46,000 daily observations back to 2018**, spanning the rating spectrum from A (Malaysia) through AA/BBB (Saudi Arabia, Indonesia) to BB/B high yield (Türkiye, Bahrain). These daily G-spreads are Cbonds' model-implied fair values, not trade prints: we verified that no continuous trade-level series exists (these sukuk print only a handful of times per quarter), so the model series is the only — and the standard — continuous data for instruments this illiquid. A single bond's series cannot be used directly: as the bond ages its maturity shortens, so its spread slides down the term structure, and this *roll-down* is a deterministic trend that masquerades as mean-reversion. We remove it by reconstructing, for each issuer and each month, the spread at *fixed* tenors (3, 5 and 7 years) from a fit to the cross-section of that issuer's outstanding bonds, and estimate dynamics on the constant-maturity series. We fit a discrete Ornstein–Uhlenbeck / AR(1) process, $\hat{\kappa}_{t+1} = c + \phi\,\hat{\kappa}_t + \varepsilon_{t+1}$ with $\alpha = -(\ln\phi)/\Delta t$, $\Delta t = 1/12$ (annualised), long-run mean $\bar{\kappa} = c/(1-\phi)$ and half-life $12\ln 2/\alpha$ months, as in eq. (9), applying Kendall's finite-sample bias correction to ϕ and Newey–West standard errors to α.

Results. The riba-free intensity is strongly mean-reverting on real data (Figure 3, Table 7):

1. **Long-run level, by credit quality.** At the 7-year tenor the long-run mean $\bar{\kappa}$ ranges from 0.9% per annum for Malaysia through 2.3% (Saudi Arabia) and 2.8% (Indonesia) to 5.2% for Bahrain, with Türkiye at $\approx$ 7% (5-year). The investment-grade issuers cluster at $\bar{\kappa} \approx$ 1–3% and the high-yield issuers at $\approx$ 5–7% — a clean three-to-five-fold separation that matches both the cross-section (Section 5.1) and the order of magnitude (10^{-2} p.a.) the credit channel predicts, now recovered from an independent, dynamic estimation. (The fine ordering carries the same two wrinkles as the cross-section: Malaysia, a tight and infrequent issuer, prints below higher-rated Saudi Arabia, and Türkiye's level is lifted by its 2022–2024 lira-crisis window; the broad IG-versus-HY split is unaffected.)
2. **Mean-reversion.** On the longest, best-sampled series — the 7-year tenor, with up to 93 monthly observations from 2018 — the bias-corrected half-life is short and broadly similar across issuers: 1.4 months (Malaysia), 2.7 (Bahrain), 3.3 (Saudi Arabia) and 4.3 (Indonesia), with α significant for the three with the longest histories (1.95–3.11, standard errors 0.51–1.27). All are *faster* than the multi-quarter-to-multi-year half-lives documented for conventional sovereign spreads in the sovereign credit literature [4] — estimated directly on real sukuk. Türkiye is the one issuer whose mean-reversion is *not* identified (α = 0.4–0.8, standard errors larger than the point estimates): its 2022–2026 sample is dominated by a single secular decline in spreads, a near-unit-root trend rather than reversion, so we report its level but not a half-life.
3. **Feller / non-negativity.** The Feller condition $2\alpha\bar{\kappa} \geq \nu^2$ holds in *all twelve* constant-maturity series (five issuers $\times$ three tenors, where sampled). The real κ process stays strictly positive under mean reversion — the structural property the paper claims, now an empirical fact across the rating spectrum rather than an artefact of a low-volatility construction.

What this settles. With the cross-section (Section 5.1) and these dynamics, the empirical content of the κ-yield curve — its level, its credit ordering, its slope, its mean-reversion and its non-negativity — rests entirely on observed sukuk. The two pieces of real evidence are an instrument-level cross-section (195 bonds, sovereign and corporate) and constant-maturity time series.

Full-universe dynamics. We extend the five-issuer dynamics to the entire outstanding USD-sukuk universe. Pulling the daily Cbonds G-spread history of all 409 outstanding sukuk (2015–2026; Cbonds account 972307) and reconstructing constant-maturity $\hat{\kappa}$ by *interpolation only*—reading a fixed tenor solely where the issuer's own bonds bracket it, never extrapolating—yields identifiable mean-reversion for **21 issuers**, four times the headline subset (Table 6). The method validates against the careful per-issuer estimates on the overlap (Saudi Arabia 2.36 vs 2.31%, Saudi Electricity 2.32 vs 2.20%, DP World 3.51 vs 3.21%), and issuers whose bonds cannot bracket a standard tenor (e.g. short-dated Islamic-bank curves) are *dropped* rather than extrapolated. Across the 21, $\bar{\kappa}$ spans 2.0% (First Abu Dhabi Bank) to 12.1% (Arada Developments) in broad alignment with credit quality — tight bank and quasi-sovereign names at the bottom, leveraged developers (Dar Al Arkan, Arada) at the top — though the alignment is broad rather than strict: Tenaga Nasional, an investment-grade utility, prints 6.4% amid high-yield names, a reminder that an interpolated single-issuer tenor can carry liquidity and curve-coverage noise. (Table 6 is sorted by $\bar{\kappa}$, not by rating; we do not claim strict monotonicity here, only the broad ordering.) The bias-corrected half-life is short throughout (median 1.8 months, monthly persistence $\hat{\phi} \approx 0.68$), and **the Feller non-negativity condition holds in all twenty-one**—so the mean-reverting, strictly-positive character of the riba-free intensity is now an empirical fact across the traded sukuk market, not a five-issuer illustration.

5.4 The κ-Index: A Cross-Sectional Benchmark Series

The estimates above price individual issuers and tenors; markets also need a *headline* number — the figure a terminal screen displays, the state variable a protocol publishes, the series a stabilisation fund watches. We define the **κ-index** as the cross-sectional median of $\hat{\kappa}$ across all quoted issuers in the full universe, computed monthly from the complete history panel (273 bonds aggregated to 37 issuer-month series with at least four years of data). The resulting series spans 127 months (December 2015 – June 2026; Figure 4) with mean 4.18% and standard deviation 1.80%. Its measured dynamics are the slow-moving common credit factor: monthly AR(1) coefficient $\rho = 0.921$ (half-life 8.4 months), innovation $\sigma = 0.66\%$ — far more persistent than any single issuer's constant-maturity series, because the median filters idiosyncratic noise. The empirical $p95$ and $p99$ of the index (7.86% and 9.48%) provide the stress thresholds used as governance constants in downstream applications, and the decade contains two stress episodes against which any κ-linked mechanism can be back-tested: March–June 2020 (peak 11.63%, a +762bp jump over base, decay half-life three months from peak) and early 2016 (+500bp, slower decay). The index requires no committee

Issuer	$\bar{\kappa}$%	HL	Issuer	$\bar{\kappa}$%	HL
First Abu Dhabi Bank	1.96	0.3	Emirates Islamic Ban	3.20	1.6
Khazanah Nasional	2.19	1.0	Indonesia	3.37	6.2
Saudi Electricity Co	2.32	0.9	DP World	3.51	1.8
Saudi Arabia	2.36	2.7	Sharjah	4.37	2.6
Saudi Arabian Oil	2.49	0.2	Bapco Energies	5.71	0.9
Saudi National Bank	2.49	0.9	Tenaga Nasional	6.40	10.6
Public Investment Fu	2.58	0.6	Bahrain	7.87	6.1
Dubai Islamic Bank	2.77	1.8	Dar Al Arkan	9.84	1.9
Aldar Investment Pro	2.78	0.5	Turkey	10.03	7.1
Majid Al Futtaim Hol	3.14	1.1	Arada Developments	12.13	2.3
Emaar Properties	3.16	2.1			

Table 6: Constant-maturity κ dynamics on the full outstanding USD-sukuk universe (interpolation-only reconstruction, validated against the five-issuer subset; Cbonds 2015–2026, $\delta = 0.60$). **21 issuers** have identifiable mean-reversion (vs. five in the headline panel); rows are sorted by $\bar{\kappa}$, which runs from 2.0% (First Abu Dhabi Bank) to 12.1% (Arada) in broad — not strict — alignment with credit quality (Tenaga Nasional, investment grade, prints 6.4%; see text), half-life (HL, months) is short throughout, and the Feller non-negativity condition holds in *all 21*. Overlap issuers reconcile with the careful per-issuer estimates (Saudi Arabia 2.36 vs 2.31; Saudi Electricity 2.32 vs 2.20; DP World 3.51 vs 3.21).

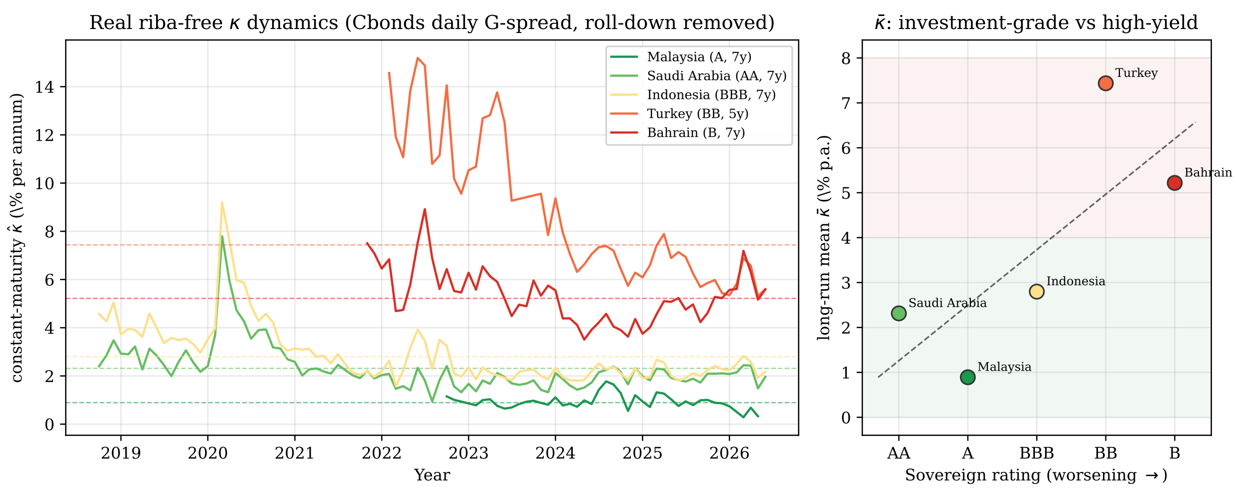

Figure 3: Real constant-maturity $\hat{\kappa}$ for five sovereign sukuk issuers across the rating spectrum, 2018–2026 (Cbonds daily G-spread, roll-down removed by fixed-tenor reconstruction; headline tenor 7y, or 5y for Türkiye). *Left:* each issuer's series mean-reverts around its own long-run level $\bar{\kappa}$ (dashed); the COVID-2020 and 2022 spikes decay back within months for the identified issuers. *Right:* $\bar{\kappa}$ separates investment grade (≈ 1–3%: Malaysia, Saudi Arabia, Indonesia) from high yield (≈ 5–7%: Bahrain, Türkiye), recovered independently from the time series. Bias-corrected 7-year half-lives are 1.4–4.3 months; the Feller non-negativity condition holds in all twelve constant-maturity series.

Issuer (rating)	Tenor	Months	$\bar{\kappa}$ (%)	Half-life (mo)	α (s.e.)	Feller
Malaysia (A)	7y	43	0.89	1.4	5.83 (3.79)	OK
Saudi Arabia (AA)	3y	43	1.42	1.3	—	OK
Saudi Arabia (AA)	5y	69	1.72	1.3	—	OK
Saudi Arabia (AA)	7y	93	2.31	**3.3**	2.53 (0.71)	OK
Indonesia (BBB)	3y	55	1.52	0.7	—	OK
Indonesia (BBB)	5y	70	1.99	1.9	—	OK
Indonesia (BBB)	7y	93	2.80	**4.3**	1.95 (0.51)	OK
Türkiye (BB)	3y	53	6.31	n.i.	0.40 (0.85)	OK
Türkiye (BB)	5y	50	7.44	n.i.	0.79 (0.87)	OK
Bahrain (B)	3y	40	4.70	2.8	—	OK
Bahrain (B)	5y	56	4.49	2.3	—	OK
Bahrain (B)	7y	56	5.22	**2.7**	3.11 (1.27)	OK

Table 7: Real constant-maturity κ dynamics for five sovereign issuers spanning the rating spectrum (Cbonds daily G-spread, monthly sampling, $\delta = 0.60$; 54 bonds). Half-lives are Kendall bias-corrected; α is the annualised mean-reversion speed with Newey–West standard error (shown for the headline 7-year series; "n.i." = not identified). $\bar{\kappa}$ is the long-run mean. Feller = $2\alpha\bar{\kappa} \geq \nu^2$, satisfied in every series. The 7-year tenor has the longest history (2018–2026) and densest curve coverage and is the most reliable; shorter tenors give downward-biased half-lives from sparser short-maturity fits. Türkiye's mean-reversion is not identified — its 2022–2026 sample is a single secular spread decline (near-unit-root), so only its level is reported. Malaysia has only a 7-year reconstruction (four bonds, all long-dated).

and no survey: it is a deterministic statistic of observed market prices, recomputable by anyone from the replication data. Its manipulation resistance should be stated precisely: the median's 50% breakdown point means the index cannot be driven *arbitrarily far* without distorting half the quoted universe, but a *bounded* shift requires only moving quotes near the middle of the cross-section — so the median is a mitigation against gross corruption, not against marginal drift, and the downstream defence rests on the change-point monitor and depth rule of the measurement-integrity companion paper rather than on the median alone.

5.5 The Benchmark-Free Estimator and the Recovery Floor

A fair objection to everything above is that $\hat{\kappa} = s/(1-\delta)$ is operationally parasitic on the conventional system: the G-spread s cannot be computed without the Treasury curve, so the "riba-free" intensity is an arithmetic residual of the riba system. The $\iota = 0$ framework, however, implies its own estimator that uses no benchmark whatsoever. At $\iota = 0$ a claim's price is the *undiscounted* expectation of its payoff at the stopping or maturity time; in zero-coupon-equivalent form, per unit face, $P(\kappa) = e^{-\kappa T} + \delta\,(1 - e^{-\kappa T})$, which inverts against the

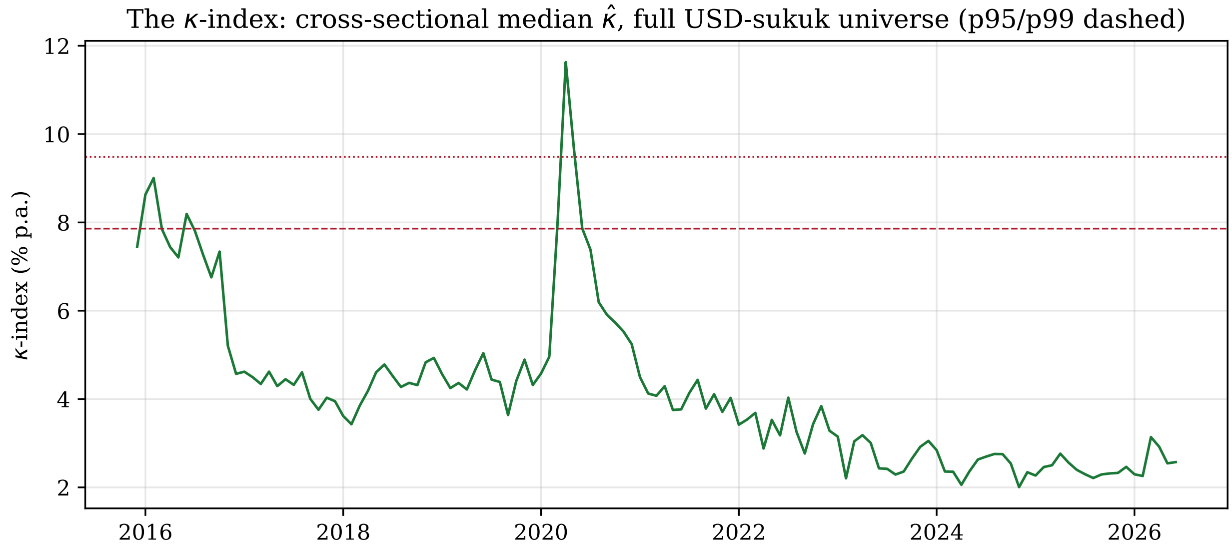

Figure 4: The κ-index: monthly cross-sectional median $\hat{\kappa}$ across the full USD-sukuk universe, December 2015 – June 2026, with the empirical $p95$ (dashed) and $p99$ (dotted) thresholds. The 2016 and 2020 stress episodes are visible; the 2020 peak is 11.63%.

yield-implied price $P = e^{-yT}$ as

$$\hat{\kappa}_{\text{price}} \;=\; -\frac{1}{T}\,\ln\frac{e^{-yT}-\delta}{1-\delta},$$

a function of the bond's *own* yield and maturity only. Three results on the full-universe snapshots (60 sovereign and 213 corporate bonds with quoted yield, spread, and maturity):

First, the cross-section is benchmark-free recoverable. The rank correlation between $\hat{\kappa}_{\text{price}}$ and the spread-based $\hat{\kappa}$ is 0.84 (sovereign) and 0.94 (corporate) at $\delta = 0.10$: the credit ordering, the curve shapes, and all relative pricing in this paper survive with the Treasury curve deleted from the pipeline.

Second, the level wedge is the reserve currency's yield layer, isolated — and we are careful about what that layer is. $\hat{\kappa}_{\text{price}}$ exceeds the spread version by a median of 458–459bp of κ — against 472bp predicted by mapping the prevailing Treasury yield through the same transformation, and the wedge tracks the Treasury yield through time (the out-of-sample companion measures $\rho = 0.93$ over 2015–26). Passed through the programme's own realised-hazard decomposition, this wedge is *not* a measured default intensity of the reserve sovereign: the US default record puts the realised intensity near zero, so the wedge is almost entirely the time-value and risk-premium layer of the reserve currency — the same layer the programme identifies as the removable one, here made visible rather than absorbed into κ. The correct statement is therefore about *orderings, not levels*: because the wedge is a common additive shift, the credit ordering, the curve shapes, and all relative pricing survive with the Treasury curve deleted (the rank correlations above), and netting against the Treasury curve is a *decomposition convenience* — it isolates issuer excess over the dollar system's base yield layer — not an operational dependency for anything cross-sectional. In a mature κ-world the netting benchmark is unnecessary for relative pricing: κ inverts from prices and cashflows alone, and whatever time or premium content that world's clearing prices carry becomes a measured quantity in the level, exactly as it is here.

Third, the framework's own consistency condition adjudicates the recovery rate. At $\iota = 0$ a price can never fall below the undiscounted recovery δ — so market prices with $e^{-yT} < \delta$ are inconsistent with the pair $(\iota = 0, \delta)$. At the conventional $\delta = 0.60$, 11–13% of observed prices in both universes violate this floor (long-maturity bonds especially); at $\delta = 0.10$ — precisely the realized recovery measured on this market's own defaulted corporate sukuk in the companion credit-protection study — feasibility is 100% and the benchmark-free correlation is at its maximum. The $\iota = 0$ pricing identity and the measured default evidence thus select the same low corporate recovery independently; under it, corporate $\hat{\kappa}$ *levels* reported above rescale by the factor $(1 - 0.60)/(1 - 0.10) \approx 0.44$ while every ordering and slope result is unchanged (the rescaling is uniform). Sovereign recoveries remain a convention — the dataset's own finding is that no sovereign USD sukuk has ever defaulted — and sovereign results are reported at the standard $\delta = 0.60$ with this caveat attached.

6 Discussion

6.1 Practical Implications for Islamic Finance

The κ-yield curve constructed in this paper has three immediate practical applications.

Sukuk pricing benchmark. Conventional sukuk pricing currently uses SOFR or government bond yields as the risk-free benchmark, creating an implicit interest-bearing component in an ostensibly Shariah-compliant product. The κ-yield curve — estimated here on observed sovereign sukuk (Sections 5.1–5.3) and, in production, extended to a full Islamic-credit panel — provides a term structure whose return object carries no interest-bearing component; in the present implementation the US Treasury curve enters only as a netting benchmark to isolate the credit component of the observed yield, and a production implementation would benchmark sukuk yields against the sovereign κ-curve itself. Portfolio managers can then strip sukuk option-adjusted spreads into a κ-term and a credit-specific component, analogous to the Z-spread decomposition in conventional bond analysis.

Islamic monetary policy. Central banks in GCC and Southeast Asian markets (Bank

Negara Malaysia and the GCC central banks, including the Saudi Central Bank, SAMA) currently operate dual monetary systems, maintaining both conventional and Islamic policy rates. The κ-yield curve offers a data-driven, no-arbitrage alternative to the administered Islamic policy rate: monetary authorities could target $\bar{\kappa}$ directly, using sovereign sukuk issuance to shift the term structure in a manner analogous to conventional open-market operations.

Halal derivatives valuation. Ahmed and Bhuyan [2] apply the AHJ framework to price everlasting options, takaful (Islamic insurance) and iCDS (Islamic credit default swaps) at $\iota = 0$. All three product families require a term structure of κ as an input. Once estimated on real sukuk spreads, κ-yield curves built with this pipeline can be fed directly into those pricing engines, replacing the ad hoc convention of using LIBOR/SOFR with an appropriate Islamic flat rate. In particular, a traded constant-maturity κ benchmark is the natural settlement reference for asset-linked risk-transfer instruments — e.g. hedging an Islamic institution's sukuk-portfolio yield exposure in place of an interest-rate swap, or pricing protection on tokenised, asset-backed positions — where the instrument's function is risk transfer rather than rate speculation; product-layer Shariah design questions for such instruments are treated in the companion Shariah-boundaries note [3].

6.2 Non-Negativity and the Negative-Rate Dilemma

κ is non-negative by construction, and the Feller condition — satisfied in every real constant-maturity series we estimate (Section 5.3) — guarantees it stays strictly positive under mean reversion. This stands in sharp contrast to the euro-area and Japanese overnight benchmark and policy rates, which were negative in the eurozone (2014–2022) and Japan (2016–2024). Negative rates create theological and structural problems for Islamic finance: if the reference rate used in sukuk pricing turns negative, it implies negative mark-ups on *murabaha* and negative profit rates on *musharaka* instruments, both of which are arguably contrary to the spirit of risk-sharing embedded in Islamic contracts.

The non-negativity guarantee of κ — derived not from a constraint but from the fundamental structure of credit pricing — means that central banks in jurisdictions operating under Islamic finance frameworks never need to navigate the negative-rate policy dilemma that bedevilled the ECB and BOJ. The κ-yield curve thus offers a practically attractive framework for Islamic monetary policy in addition to settling the riba question at the pricing layer. Product-layer Shariah questions (gharar, maysir, possession) are outside the scope of this construct and remain with Shariah supervisory boards; see the companion Shariah-boundaries note [3].

7 Conclusion

We define the κ-yield curve — the riba-free analogue of the risk-free term structure — and estimate it entirely on real traded sukuk: point identification $\hat{\kappa} = s/(1-\delta)$ on observed G-spreads, a cross-sectional characterisation by rating and maturity, and AR(1)/OU estimation of CIR mean-reversion on roll-down-free constant-maturity series. The empirical content rests on two real datasets — no synthetic panel.

Result 1 (Level and credit ordering). On a real cross-section of 195 actively-quoted USD sukuk (40 sovereign, 154 corporate, one municipal; Cbonds), the convergence intensity is of order 10^{-2} per annum and broadly increasing as credit quality deteriorates, from $\hat{\kappa} \approx 0.9\%$ at AAA to $\approx 4.4\%$ at single-B (apart from the Saudi-dominated AA bucket, which prices just above single-A) — the credit ordering the framework predicts, read directly from the sukuk market.

Result 2 (Term structure). The investment-grade κ-curve slopes upward: the G-spread rises with maturity at +1.17 bp/yr pooled, +0.63 bp/yr with rating fixed effects ($n = 164$). The slope is positive but modest — an honest real-data estimate, weaker than a designed curve.

Result 3 (Dynamics and Feller). On roll-down-free constant-maturity series for five issuers across the rating spectrum, κ mean-reverts with a bias-corrected half-life of 1.4–4.3 months at the 7-year tenor around a long-run mean that separates investment grade ($\approx$ 1–3%) from high yield ($\approx$ 5–7%), faster than conventional sovereign spreads, and the Feller non-negativity condition holds in all twelve series.

The contribution is the κ-yield-curve construct together with a real-data characterisation of its level, credit ordering, slope and dynamics on traded sukuk.

Future work. The two real datasets cover complementary dimensions — a 195-bond cross-section and constant-maturity time series (five issuers, up to eight years). The natural next steps are (i) a full real panel uniting both — every issuer, every month — to estimate the CIR dynamics at the instrument level; (ii) deepening the sovereign/corporate decomposition begun in Section 5.2 to a full sector-by-rating corporate κ surface; and (iii) construction of a κ-based Islamic yield curve model that central banks can use for open-market operations targeting the term structure of the convergence intensity.

Data Availability Statement

All empirical results in this study are estimated on real traded sukuk from the Cbonds bond database (https://cbonds.com). The cross-section (Section 5) is the set of outstanding, actively-quoted USD sukuk retrieved live from the Cbonds Bond Screener (/api/bonds/search/): the 195-bond sovereign-plus-corporate panel is shipped as real_sukuk_panel_full.csv and the per-rating / term-structure tables and the κ-curve (Figure 1) are reproduced from it by real_panel_analysis.py; the 61-bond sovereign snapshot (issuer, ISIN, maturity, yield, G-spread, rating) is shipped as sukuk_cbonds_panel.csv, and Figure 2 is reproduced from it by sukuk_cbonds_panel.py and fig_sukuk_cbonds_curve.py. The maturity-matched US Treasury benchmark underlying the G-spread is Cbonds' interpolation of the public UST par curve. SOFR is from FRED (https://fred.stlouisfed.org).

The real κ dynamics of Section 5.3 use the daily G-spread histories (2018–2026) of all 54 USD sovereign sukuk of the five issuers (Indonesia, Saudi Arabia, Bahrain, Türkiye, Malaysia), retrieved from the same Cbonds source. The extracted daily histories and the derived constant-maturity series are shipped as sukuk_curve_histories_all.json and sukuk_cm_series.csv; the constant-maturity construction, the Kendall-bias-corrected AR(1)/OU estimation and Figure 3 are reproduced from those files by sukuk_constant_maturity.py. The instrument-level sukuk-versus-conventional comparison (USD) and the sovereign/corporate decomposition (Section 5.2) draw on, respectively, the 133 conventional sovereign USD bonds and 213 corporate USD sukuk of the same issuers, also from Cbonds; both extracts ship as sukuk_vs_conventional.csv and corporate_sukuk_panel.csv. No Cbonds subscription is needed to reproduce any reported number from the shipped extracts.

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A Derivation of the Identification (Theorem 2.1)

We outline the derivation of equation (2) from the reduced-form no-arbitrage conditions at $\iota = 0$.

Consider protection on a unit notional with loss-given-default $(1-\delta)$ at the default time τ_D (intensity λ under the risk-neutral measure; the reduced-form framework of 10). With discounting at the convergence intensity κ ($\iota = 0$), the protection leg is worth, at constant intensities,

$$V_{\text{prot}} = (1-\delta)\,\frac{\lambda}{\kappa+\lambda}, \tag{10}$$

and a premium leg paying the spread s continuously until τ_D is worth $s/(\kappa+\lambda)$. Equating the two legs, the common factor $1/(\kappa+\lambda)$ cancels, giving the credit triangle

$$s = \lambda(1-\delta). \tag{11}$$

The identification of the convergence intensity with the default intensity, $\kappa = \lambda$, is *not* an independent steady-state fact: it is the Random Stopping-Time Equivalence of Ahmed & Bhuyan [2] (the Separation-Theorem isomorphism between the AHJ stopping time and the reduced-form default time). Substituting yields

$$s = \kappa\,(1-\delta) \quad\Longrightarrow\quad \kappa = \frac{s}{1-\delta}. \tag{12}$$

Equivalently, the fixed-maturity survival claim of Theorem 2.1 can be priced directly: $V(t) = \mathcal{P}e^{-\kappa(1-\delta)(T-t)}$, whose continuously-compounded spread over the default-free benchmark is $s = \kappa(1-\delta)$. This completes the derivation of (2).

B GMM Moment Conditions

The two-step GMM estimator minimises:

$$Q_T(\boldsymbol{\theta}) = \mathbf{g}_T(\boldsymbol{\theta})^\top \hat{\mathbf{W}}^{-1}\mathbf{g}_T(\boldsymbol{\theta}), \tag{13}$$

where $\mathbf{g}_T(\boldsymbol{\theta}) = T^{-1}\sum_{t=1}^{T} \mathbf{z}_t \otimes \mathbf{m}_t(\boldsymbol{\theta})$, with moment conditions:

$$m_{1t}(\boldsymbol{\theta}) = \hat{\kappa}_{t+1} - \hat{\kappa}_t e^{-\alpha h} - \bar{\kappa}(1 - e^{-\alpha h}), \tag{14}$$

$$m_{2t}(\boldsymbol{\theta}) = m_{1t}^2 - \frac{\nu^2}{\alpha}\hat{\kappa}_t(e^{-\alpha h} - e^{-2\alpha h}) - \frac{\nu^2\bar{\kappa}}{2\alpha}(1 - e^{-\alpha h})^2, \tag{15}$$

and $\hat{\mathbf{W}}$ is the Newey–West HAC estimator of the long-run variance of $\mathbf{z}_t \otimes \mathbf{m}_t$.

C Country Coverage and Sukuk Universe

Table 8 summarises the real sukuk universe underlying our cross-section — the 195 actively-quoted USD sukuk by issuer country.

Table 8: Real Sukuk Universe (195 actively-quoted USD sukuk, Cbonds live pull)

Country	Sovereign	Corporate	Total	G-spread range (bp)
Saudi Arabia	12	74	86	11–264
United Arab Emirates	0	46	46	44–248
Indonesia	21	0	21	26–294
Kuwait	0	13	13	27–523
Qatar	1	11	12	29–393
Malaysia	4	7	11	13–1281
Others (HK, LU, BH, US, PH, IE)	2	4	6	32–141
Total	40	155	195	—

Notes: All instruments are outstanding, actively-quoted USD sukuk retrieved live from the Cbonds Bond Screener (account 972307), enumerated per instrument in the shipped `real_sukuk_panel_full.csv`. The constant-maturity dynamics (Section 5.3) use daily G-spread histories of 54 sukuk from five issuers, 2018–2026. Ratings are foreign-currency composites as of the pull date. The non-sovereign column (155) comprises 154 corporate sukuk and one municipal sukuk, the latter grouped with corporates here.