The κ-Framework: A Riba-Free Foundation for Islamic Monetary Systems

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Abstract

We propose the **κ-rate** — the convergence intensity of perpetual futures contracts [2] — as the first mathematically rigorous, no-arbitrage-grounded, riba-free alternative to the risk-free interest rate r in Islamic monetary economics.

The κ-rate is: (i) *endogenous* — identified from market data (the perpetual-futures basis or funding dynamics for traded contracts, the credit spread for sukuk and credit instruments), with no interest rate as an input; (ii) *observable* — computable from market observables and publishable on-chain in real time; (iii) *riba-free* — an event intensity, not a reward for the time value of money; and (iv) *complete* — sufficient to construct a full term structure (the κ-yield curve) analogous to the conventional yield curve.

We develop the theoretical foundations in four steps. First, we decompose the classical interest rate r (Fisher [27], Wicksell [49], Keynes [37]) into its components and identify the riba component as the pure time-preference premium; combining the Ackerer-Hugonnier-Jermann no-arbitrage theorem ($\iota = 0$ is arbitrage-free) with our identification of ι as that premium shows it is eliminable. Second, we show that the κ-rate is the Wicksellian "natural rate" (a rate grounded in real economic dynamics rather than monetary policy) given rigorous, riba-free form for the first time: κ plays the natural rate's role without its time-preference content. Third, we construct the κ-yield curve as the Islamic analog of the CIR [21] term structure, grounded in exponential stopping time distributions; estimated from sovereign sukuk data it is upward-sloping, exhibiting a term premium without any interest component. Fourth, we demonstrate that κ can serve all monetary functions currently performed by r in the Islamic context: pricing credit instruments (Section 6.2), anchoring actuarial takaful contributions (Section 6.3), guiding Islamic monetary policy (Section 6.4), and providing a sovereign benchmark rate for sukuk issuance (Section 6.5).

Empirical illustration spans three independent domains. The κ-yield curve is estimated *entirely on real traded sukuk*: a 195-instrument USD-sukuk cross-section (40 sovereign, 154 corporate, 1 municipal; Cbonds) in which $\hat{\kappa}$ broadly increases as credit quality declines ($\approx 0.9\%$ at AAA to $\approx 4.4\%$ at single-B, apart from the Saudi-dominated AA bucket pricing just above single-A) with an upward-sloping investment-grade term structure, plus roll-down-free constant-maturity $\hat{\kappa}$ dynamics for five issuers (mean-reverting, half-lives 1.4–4.3 months for the four identified issuers at the 7-year tenor, Feller-positive throughout). For India, the model-implied $\hat{\kappa} = 12.06\%$ is numerically close to the PMFBY crop-insurance actuarial rate ($\approx 12\%$) — a units-level coincidence, not a strict validation. Finally, dYdX v4 — an independent perpetual exchange operating at $\iota = 0$ on mainnet since 2023 — provides a commercial-scale existence proof and a *decoupled*, interest-free measurement of κ. Across domains κ recurs as the same *type* of riba-free intensity; its *magnitude* is regime-specific (fast for liquid perps, slow for credit and insurance), and we claim no numerical equality. Contemporary jurisprudential opinion [44] accepts that the $\iota = 0$ construction removes the *riba* objection specifically; *gharar* and ownership questions fall to the contract built on the rate, not the rate itself.

Keywords: κ-rate, riba-free monetary policy, Islamic monetary economics, natural rate, perpetual contracts, yield curve, Wicksell, Fisher decomposition, no-arbitrage, $\iota = 0$, sovereign sukuk, takaful

JEL Classification: E12, E43, E58, G12, G13, G15, P40.

1 Introduction

The interest rate r is the foundational pricing parameter of modern finance. It appears in every discount factor, every bond yield, every option formula, every term structure model. It is the rate at which future cash flows are brought to present value. Islamic finance prohibits it.

This is not a peripheral prohibition. The Quranic injunction against *riba* — "Allah has permitted trade and forbidden *riba*" (Al-Baqarah 2:275) — is absolute, applying to all forms of predetermined excess in monetary exchange. The risk-free rate r is, under any classical decomposition (Fisher [27], Böhm-Bawerk [19], Keynes [37]), precisely the price of time preference in monetary form: the lender receives r per unit time merely for parting with money, independent of any real economic outcome. This is *riba al-nasi'ah* — the prohibited excess for deferment — by all four Sunni schools and the Ja'fari school [47, 25].

The consequence for Islamic finance is severe. Without r, there is no discount factor. Without a discount factor, bonds cannot be priced. Without bond pricing, there is no credit market. Without a credit market, Islamic institutions cannot hedge credit risk, sovereign governments cannot issue benchmarked sukuk, and takaful operators cannot price contributions actuarially. The entire credit infrastructure of modern finance — a market of roughly \$130 trillion in bonds and \$10 trillion in credit derivatives [15] — is inaccessible to Islamic participants on Shariah-compliant terms.

Islamic monetary economists have recognised this problem for five decades (Zarqa [51], Chapra [20], Khan & Mirakhor [38], Iqbal & Mirakhor [32]), and the risk-management and securitization literatures have catalogued its consequences for Islamic credit and hedging instruments (Khan & Ahmed [39], Jobst [35]). The proposed solutions — profit-sharing rates (musharakah/mudarabah returns), commodity benchmarks, and asset-backed yield proxies — have all failed in practice. Iqbal & Molyneux [31] document that even in the most advanced Islamic finance jurisdictions (Malaysia, Bahrain, UAE), rates charged on Islamic products track LIBOR/SOFR closely, with the interest content merely re-labelled. This persistent dependence — Islamic instruments priced against conventional rates even when riba is explicitly prohibited — is widely documented in the sukuk-pricing literature [46], which notes that analysts use LIBOR or the Islamic interbank benchmark rate as the ad-hoc benchmark to evaluate sukuk despite the Shariah objection.

The failure has a structural cause: *no prior work has derived a complete, no-arbitrage-grounded alternative to r from first principles.*

This paper provides that alternative. We call it the **κ-rate**: the convergence intensity of a perpetual futures contract, proved by Ackerer, Hugonnier & Jermann [2] to achieve unique no-arbitrage pricing at $\iota = 0$ (zero interest parameter). The κ-rate is not a label change. It is a mathematically distinct pricing parameter with provably no riba content, capable of performing all the monetary functions currently assigned to r in an Islamic economy.

The Gap in One Sentence

Wicksell [49] proposed in 1898 that the "natural rate" of interest should be grounded in real economic productivity rather than monetary convention — but it remained an interest rate with time-preference content. Chapra [20] proposed in 1985 that Islamic monetary policy needs a real-activity-based pricing signal — but never provided a mathematical formula. The κ-rate realizes the Wicksellian aspiration — a real-activity-grounded rate — in a form that is rigorous, riba-free, and on-chain observable, 128 years after the question was first posed.

Structure

Section 2 decomposes r into components and identifies the riba component. Section 3 defines the κ-rate from first principles. Section 4 contrasts κ with r across all standard properties. Section 5 constructs the κ-yield curve. Section 6 develops applications: credit instruments, takaful, monetary policy, and sovereign benchmarking. Section 7 presents empirical evidence across three independent domains: sovereign sukuk markets, agricultural insurance, and the dYdX v4 perpetual exchange. Section 8 provides the Shariah analysis. Section 9 compares κ with prior proposals and discusses limitations.

2 Decomposing r: What Is the Riba Component?

2.1 The Fisher Decomposition

Fisher [27] established the modern theory of interest as an equilibrium between two forces:

1. **Time preference (impatience):** the subjective preference of individuals for present over future consumption, independent of any investment opportunity.
2. **Investment opportunity:** the marginal productivity of capital — the real return from deploying resources in production.

The equilibrium interest rate r equates marginal impatience to marginal productivity. Fisher's formulation also yields the famous Fisher equation:

$$r_{\text{nominal}} = r_{\text{real}} + \pi \tag{1}$$

where π is expected inflation. The market rate therefore contains three components: real productivity, time preference, and inflation compensation.

2.2 Böhm-Bawerk's Distinction

Böhm-Bawerk [19] distinguished two grounds for interest:

- The **productivity ground**: present capital generates future output exceeding the present input (roundabout production). This is the return to real productive activity.

- The **time preference ground**: present goods are valued more than future goods by an "agio" (premium) for their presentness — independent of productivity.

Islamic economics accepts the productivity ground (profit from real activity is halal) but rejects the time preference ground (a predetermined premium for the passage of time, payable on a money loan regardless of economic outcome, is riba). This is the precise jurisprudential boundary.

2.3 The Riba Component

Under the Fisher-Böhm-Bawerk decomposition, the risk-free interest rate r can be written:

$$r = \underbrace{r_{\text{prod}}}_{\text{halal}} + \underbrace{r_{\text{pref}}}_{\text{riba}} + \underbrace{\pi}_{\text{neutral}} \tag{2}$$

The riba component r_{pref} is the pure time preference premium: the predetermined, fixed return on a money loan attributable solely to the passage of time. It is independent of:

- the real economic outcome of the borrower,
- the productive use of the capital,
- market conditions at the time of repayment.

Under El-Gamal's [25] operationalization of riba as "predetermined excess in exchange, independent of economic outcome," r_{pref} is riba by definition. The classical Islamic foundation of this identification — money as a measure that may not itself bear a time charge, after al-Ghazālī, Ibn Taymiyyah, Ibn Khaldūn and al-Maqrīzī — is developed in the companion study [11].

2.4 The Wicksell Natural Rate and Its Limit

Wicksell [49] proposed the "natural rate" of interest r^* — the rate at which investment equals saving in a real economy, determined by the marginal productivity of capital without monetary distortion:

$$r^* = \text{MPK} = \frac{\partial F(K, L)}{\partial K} \tag{3}$$

This is closer to the Islamic ideal: a rate grounded in real activity rather than monetary policy. But Wicksell's natural rate still includes time preference: it is still an interest rate that compensates capital for time elapsed, just one calibrated to real productivity. The riba component persists.

Remark 1. The κ-rate achieves what Wicksell attempted but could not complete. It eliminates r_{pref} entirely from the pricing formula, leaving only the productivity-equivalent component in the form of convergence dynamics. In this structural sense — developed further in the Discussion — the κ-rate plays the role of Wicksell's natural rate with the riba removed, though it is not numerically identical to the marginal product of capital.

Structure versus magnitude (a caveat carried throughout). The decomposition (2) is conventional theory's accounting of r; the κ-rate's riba-free property is a statement about its *generating process*—identified from the stopping intensity of a real claim, with no predetermined time-preference term—not about its numerical magnitude. A calibrated $\hat{\kappa}$ extracted from markets that still price r inherits those magnitudes, so κ and r may coincide numerically; the companion yield-curve study's benchmark-free estimator reads the entire observed yield as event intensity for exactly this reason. What makes κ riba-free is therefore structural and outcome-contingent, and a real-only *magnitude* is realized only as markets come to price κ directly—a post-mainnet property, not a present one.

3 The κ-Rate: Definition and Properties

3.1 From Perpetual Futures to Monetary Rate

We recall the foundational result. Ackerer, Hugonnier & Jermann [2] prove that the unique no-arbitrage price of a perpetual futures contract on a spot asset X_t (under constant parameters) is:

$$f_t = \frac{\kappa - \iota}{\kappa + r_b - r_a} \cdot X_t \tag{4}$$

where $\kappa > 0$ is the **convergence intensity**, $\iota \geq 0$ is the interest parameter, and r_a, r_b are the long and short funding rates. Theorem 3 of Ackerer et al. proves that $\iota = 0$ satisfies no-arbitrage.

κ is, primarily, the intensity of a stopping time. The economic content of κ is not the price ratio in (4) but the object that ratio reflects: κ is the intensity of the random time $\theta_t \sim t + \text{Exp}(\kappa)$ at which the contract resolves to its underlying. This is the same mathematical role played by the hazard rate λ in reduced-form credit models (Section 3.5). Defining κ as a stopping-time intensity — rather than as a particular price ratio — is what allows the single parameter to govern perpetuals, sukuk, takaful, and credit protection alike, and it is what we calibrate to data.

3.2 Identifying κ from Data: Three Channels

Because κ is an intensity rather than a quoted price, it is *identified* from whatever observable a given market provides. Three channels arise, each appropriate to a different instrument class and data environment. They estimate the same underlying object through different observables.

Channel 1 — Credit-spread identification (hazard channel). For credit instruments (sovereign sukuk, iCDS, takaful), κ is the loss-event intensity and is identified from the observed spread s and recovery rate δ by the standard reduced-form relation

$$\boxed{\hat{\kappa} = \frac{s}{1-\delta}} \tag{5}$$

This is the identification used throughout the empirical κ-yield-curve estimation [5] (Section 7.1), and it follows directly from Corollary 1: at $r = 0$, the Duffie–Singleton survival price reduces to $\exp(-\kappa(1-\delta)(T-t))$, so the spread compensates for $\kappa(1-\delta)$ and (5) inverts it. No interest rate enters.

Channel 2 — Perpetual basis identification (nonzero-basis regime). When a perpetual futures market exhibits a nonzero basis ($r_a \neq r_b$, so $f_t \neq X_t$), inverting (4) at $\iota = 0$ identifies κ from the two observed prices:

$$\hat{\kappa}_t = \frac{f_t/X_t \cdot (r_b - r_a)}{1 - f_t/X_t}, \qquad r_a \neq r_b. \tag{6}$$

This channel is available only when the basis is nonzero; it is the natural identification for markets that quote asymmetric funding.

Channel 3 — Funding-decay identification (zero-basis regime). A perpetual market may run symmetric funding ($r_a = r_b$), in which case (4) gives the parity $f_t = X_t$ and the basis formula (6) becomes indeterminate (0/0): with no basis, there is no price spread to invert. Here κ is instead identified from the *dynamics* of the funding rate. Modeling the basis $b_t = f_t - X_t$ as a mean-reverting (Ornstein–Uhlenbeck) process with reversion speed κ, the conditional mean decays at that rate,

$$\mathbb{E}[\, f_{t+\Delta} - X_{t+\Delta} \,|\, \mathcal{F}_t] = e^{-\kappa\,\Delta}\,(f_t - X_t), \tag{7}$$

so κ is recovered as the estimated mean-reversion rate (equivalently, from the half-life $\ln 2/\kappa$ of basis deviations) of the realized funding series. This is the channel relevant to symmetric-funding venues such as dYdX v4 (Section 7.2).

Remark 2 (Why three channels and not a contradiction)**.** The parity $f_t = X_t$ (zero basis) and the basis inversion (6) are not in conflict: they describe different regimes. When the basis is nonzero, κ is read from its size (Channel 2); when the basis is zero, κ is read from how fast deviations decay (Channel 3). The two perpetual channels are limits of one another — as the basis shrinks to zero, static inversion hands off to dynamic decay — and both recover the same stopping-time intensity that Channel 1 obtains from credit spreads. The constant-κ exposition here is extended to stochastic κ_t in Appendix A.

3.3 Formal Definition

Definition 1 (κ-Rate)**.** The **κ-rate** for an instrument linking a financial claim to an underlying real asset X_t is the convergence intensity $\kappa > 0$ that:

1. Parameterizes the distribution of the stopping time $\theta_t \sim t + \mathrm{Exp}(\kappa)$ at which the claim is settled.
2. Is identified endogenously from market data via the appropriate channel of Section 3.2 (credit spread, perpetual basis, or funding decay), with no interest rate as an input.
3. Satisfies $\iota = 0$: no interest parameter is charged.
4. Achieves unique no-arbitrage pricing by Theorem 3 of Ackerer et al. [2] and, for credit instruments, by Corollary 1, in the martingale sense of Harrison & Kreps [28]: pricing proceeds under an equivalent measure with no reference to a time-preference discount.

3.4 Economic Interpretation of κ

κ measures the **speed of convergence** between a financial instrument's price and its underlying real asset value. The interpretations below are clearest in the nonzero-basis regime (Channel 2), where the basis is observable; in the zero-basis regime they are read equivalently from the decay rate (7):

- **High κ:** Rapid mean-reversion. The perpetual price tracks spot closely. Short expected stopping time $\mathbb{E}[\theta_t - t] = 1/\kappa$. The market is confident about near-term convergence.
- **Low κ:** Slow convergence. Wide basis. Longer expected stopping time. The market perceives structural uncertainty about when the real asset will reach its fair value.
- **$\kappa \to \infty$:** Instantaneous convergence; deviations decay immediately and $f_t = X_t$. Perfect tracking.
- **$\kappa \to 0$:** Deviations do not decay; the perpetual becomes untethered from the underlying and the instrument loses economic meaning.

This interpretation is fundamentally different from r:

Property	**r (interest rate)**	**κ (κ-rate)**
Determined by	Central bank	Market prices
Measures	Time preference	Convergence speed
Riba content	Yes ($r_{\text{pref}} > 0$)	No
Real basis	Partial	Complete
Negative values	Possible ($r < 0$)	Impossible ($\kappa > 0$)
Observable	Policy setting	On-chain, real-time

3.5 The Duffie–Singleton Isomorphism

The Ackerer framework prices perpetual contracts, but the κ-rate's applicability extends much further. The bridge is a formal isomorphism between the Ackerer perpetual pricing framework and the Duffie–Singleton [23] reduced-form credit risk model, which we now establish. Credit-risk pricing has two classical strands: the *structural* approach, which models default as the firm's asset value crossing a boundary (Merton [41]), and the *reduced-form* approach, which treats default as the first jump of a point process with an exogenous hazard rate (Jarrow & Turnbull [34], Lando [40], Duffie & Singleton [23]). Both strands embed the risk-free rate r as a discount factor; our isomorphism shows that, in the reduced-form case, this r is the discount component that the riba-free case ($\iota = 0$) eliminates.

In the Duffie–Singleton model, a defaultable bond is priced as:

$$P(t,T) = \mathbb{E}_t\left[\exp\left(-\int_t^T (r_s + \lambda_s(1-\delta_s))\,ds\right)\right] \quad (8)$$

where λ is the default hazard rate, δ is the recovery rate, and r is the risk-free rate. Default occurs at the random stopping time τ with intensity λ.

Proposition 1 (Isomorphism)**.** *The Ackerer perpetual pricing framework and the Duffie–Singleton reduced-form credit framework are formally isomorphic under the identification:*

Ackerer (perpetual)	$\leftrightarrow$	***Duffie–Singleton (credit)***
Convergence intensity κ	$\leftrightarrow$	*Default hazard rate* λ
Interest parameter ι	$\leftrightarrow$	*Risk-free rate* r
Stopping time θ_t	$\leftrightarrow$	*Default time* τ
Perpetual contract	$\leftrightarrow$	*Defaultable bond*

Proof sketch. Both frameworks model an exponential stopping time and a conditional payoff at that time. In the Ackerer framework, the stopping time θ_t has conditional density $q(\theta_t \in ds \mid \mathcal{F}_t) = (\kappa_s - \iota_s)\exp(-\int_t^s(\kappa_u - \iota_u)\,du)\,ds$. In the Duffie–Singleton framework, the default time τ has hazard rate λ and conditional survival probability $\exp(-\int_t^T \lambda_s\,ds)$. Under the bijection $\kappa \leftrightarrow \lambda$, $\iota \leftrightarrow r$, $\theta_t \leftrightarrow \tau$, the survival/stopping structure and the no-arbitrage condition of one framework map term-by-term to the other: any pricing equation valid in one yields a valid pricing equation in the other under substitution. The recovery rate δ enters only in the payoff at the stopping time (a defaulted bond pays δ; an iCDS pays $1-\delta$), so the credit *spread* is $s = \kappa(1-\delta)$ while $\kappa \equiv \lambda$ is the intensity itself — the relation inverted in the empirical identification (5). The detailed proof is given in Ahmed & Bhuyan [4]. □

Corollary 1 ($r = 0$ in Credit Pricing)**.** *No-arbitrage at $\iota = 0$ in the Ackerer framework implies no-arbitrage at $r = 0$ in the Duffie–Singleton credit framework. The stopping time distribution alone is sufficient to carry timing information for fair credit pricing.*

The corollary is the operative result. Theorem 3 of Ackerer et al. proves no-arbitrage at $\iota = 0$ for perpetuals; by the isomorphism, no-arbitrage at $r = 0$ holds for credit instruments. This is the formal bridge from the κ-rate as a perpetual pricing parameter to the κ-rate as a complete riba-free foundation for sukuk, takaful, and credit derivatives. All of the applications developed in Section 6 rest on Proposition 1 and Corollary 1.

Remark 3 (The Separation Theorem)**.** The conjunction of Theorem 3 of Ackerer et al. and Proposition 1 yields what may be called the *Separation Theorem*: *at $\iota = 0$ the no-arbitrage price carries no interest term, so interest is an* eliminable *component of pricing wherever contract resolution occurs at a random stopping time — the stopping-time distribution then carries all timing information previously attributed to the discount factor.* Away from $\iota = 0$ the parameter is not a separable discount: in the AHJ random-time representation it enters the stopping intensity itself, $\theta \sim \mathrm{Exp}(\kappa - \iota)$, so the theorem asserts that the riba-free point is well-defined and arbitrage-free, not that interest factors out for all ι. The empirical corroboration is the European negative-rate environment of 2014–2022, during which credit markets priced risk efficiently at $r \le 0$ — confirming that the discount factor and the no-arbitrage condition are not coextensive.

4 κ vs. r: A Complete Comparison

4.1 The Riba Test

El-Gamal's [25] criterion for riba: a payment is riba if it is (1) predetermined, (2) in excess of principal, and (3) independent of real economic outcome.

Proposition 2 (The κ-Rate is Not Riba)**.** *The κ-rate satisfies none of the three riba criteria:*

1. ***Not predetermined.*** *κ_t fluctuates continuously with market conditions. It is implied from market prices, not contractually fixed.*
2. ***Not excess of principal.*** *The κ-rate does not generate a return over principal. At $\iota = 0$ and $r_a = r_b$, the instrument price equals the underlying asset value ($f_t = X_t$) — no excess.*
3. ***Not independent of economic outcome.*** *κ is backed out from the observed basis f_t/X_t, which reflects real market views about asset convergence. If the underlying asset performs well, the basis narrows (high κ); if poorly, the basis widens (low κ). The κ-rate is tied to real economic dynamics.*

Proof. (1) follows from the identification channels of Section 3.2: $\hat{\kappa}_t$ is a continuous function of market observables (prices or funding dynamics), which vary. (2) follows from the parity established in Section 3.2: at $\iota = 0$ and $r_a = r_b$, equation (4) gives $f_t = X_t$. (3) follows from the market-determination of $\hat{\kappa}_t$: it is identified from observed data, not set exogenously. □

Remark 4 (Scope of criterion (2))**.** The parity argument for criterion (2) ($f_t = X_t$, no excess) is the zero-credit-risk case ($\iota = 0$, $r_a = r_b$). For a credit instrument the fair spread $\kappa(1-\delta) > 0$ is not an excess over principal but actuarial compensation for an expected loss—Paper 9's cost-of-capital identity decomposes the required return as a productivity/time-value term plus exactly $\kappa(1-\delta)$. Because El-Gamal's criteria are conjunctive (a payment is riba only if it is predetermined *and* in excess *and* outcome-independent), criteria (1) and (3)—which hold for credit instruments without qualification—are each individually sufficient to defeat the charge. The parity leg is therefore redundant for the credit case, not load-bearing: the outward-facing defense leads with outcome-contingency (criteria 1 and 3) and the portfolio-level rebuttal of the companion safe-asset study, never with parity.

4.2 Replacing r in Standard Pricing Formulas

The key question is whether κ can perform the mathematical functions of r. We demonstrate this across three standard settings.

Bond pricing. A conventional zero-coupon bond: $P = e^{-rT}$. The κ-equivalent (Islamic perpetual sukuk with expected maturity $T = 1/\kappa$):

$$P^{\kappa} = \mathbb{E}^{Q}[X_{\tau}], \quad \tau \sim \mathrm{Exp}(\kappa) \tag{9}$$

(equal to X_t in the martingale case). The discount comes not from e^{-rT} but from the stopping time distribution: the probability that the redemption event is more than T periods away decays exponentially at rate κ, achieving the same economic effect without any interest.

Option pricing. In Black-Scholes [18], the risk-free rate r appears as the discount factor and in the drift of the risk-neutral measure. At $\iota = 0$ and $r_a = r_b$, the everlasting put from Ackerer et al. Proposition 6 (for $x \geq K$) is

$$\Pi(x, K) = \frac{K^{1-\beta_p}}{\beta_c - \beta_p} \cdot x^{\beta_p}, \quad \beta_{p,c} = \tfrac{1}{2} \mp \sqrt{\tfrac{1}{4} + \tfrac{2\kappa}{\sigma^2}} \tag{10}$$

(with the intrinsic value $K - x$ added where $x < K$), replacing r with κ entirely in the exponent.

Credit default swap. A conventional CDS spread satisfies $s \approx \lambda(1 - \delta)$, the hazard rate times loss-given-default. The iCDS spread [7] is $s^* = \kappa(1 - \delta)$ with $\kappa \equiv \lambda$ and $\iota = 0$, achieving the same risk transfer without any interest baseline (Section 6.2).

4.3 Policy Functions of r and Their κ Analogs

Table 1 lists the monetary policy functions of r and the corresponding κ-rate mechanism.

Table 1: Monetary functions: r vs. κ.

Function	r mechanism	κ mechanism
Price of credit risk	$r + \lambda$ spread	$\kappa \equiv \lambda$ at $\iota = 0$
Discount factor	e^{-rT}	$\mathrm{Exp}(\kappa)$ survival prob.
Term structure	Yield curve $r(T)$	κ-term-structure (upward)
Monetary signal	Central bank sets r	Market implies $\hat{\kappa}$
Sovereign benchmark	Government bond yield	Perpetual sukuk implied κ
Insurance pricing	$e^{-r} \cdot$ claim	$\pi^* = \kappa B$

5 The κ-Yield Curve

5.1 Construction

The conventional yield curve $r(T)$ maps maturity T to the yield on a zero-coupon risk-free bond. It is the term structure of interest rates, grounded in the expectation hypothesis and risk premia over time. Cox, Ingersoll & Ross [21] provide the canonical equilibrium derivation; Vasicek [48] provides the Gaussian version; Nelson & Siegel [43] provide the empirical estimation framework.

The **κ-yield curve** is the Islamic analog: the mapping $\kappa(T)$ of convergence intensities from perpetual instruments of different expected horizons.

Proposition 3 (Horizon–Intensity Identity)**.** *For a single instrument whose stopping event has expected horizon T (the expected time until resolution under no other information), the convergence intensity and the horizon are reciprocal:*

$$\kappa(T) = \frac{1}{T}, \qquad T > 0. \tag{11}$$

A short expected horizon corresponds to a high intensity (rapid convergence); a long expected horizon to a low intensity.

Proof. For a stopping time $\theta_t \sim \mathrm{Exp}(\kappa)$, the expected stopping time is $\mathbb{E}[\theta_t - t] = 1/\kappa$. Setting this equal to the instrument's expected horizon T gives $\kappa = 1/T$. □

Remark 5 (The identity is not the term structure)**.** Equation (11) relates one instrument's intensity to its own expected horizon; it is a definitional reciprocal, not a claim about how κ varies across the maturity spectrum of a given issuer. The latter object — the empirical κ-*term-structure* for a fixed credit, estimated from sukuk of different maturities in Section 7.1 — is *upward-sloping*: longer-dated claims on the same sovereign carry higher cumulative credit uncertainty and hence higher κ. There is no contradiction: (11) maps horizon to intensity for a single contract, whereas the empirical term structure maps maturity to intensity across contracts on one issuer. We reserve "κ-yield curve" for the latter, empirically estimated, object in what follows.

5.2 Comparison with the Conventional Yield Curve

Table 2 contrasts the two term structures.

5.3 Empirical Estimation

By Proposition 3, to estimate the κ-yield curve one needs:

Table 2: κ-yield curve vs. conventional yield curve.

Property	**CIR/Vasicek**	**κ-curve**
Pricing parameter	Short rate r_t	Conv. intensity κ_t
Riba content	Yes	No
Central bank role	Sets r	None (market-implied)
Shape	Any	Upward-sloping (empirical)
Calibration	Rate expectations	Credit spreads / basis
Negative rates	Possible	Impossible
DeFi observable	No	Yes (on-chain)

1. Observed perpetuals basis $f_t^{(i)}/X_t^{(i)}$ for instruments $i = 1, \ldots, n$ with different underlying asset horizons.
2. Implied $\hat{\kappa}_t^{(i)}$ from the basis channel (6) where the basis is nonzero, or from the funding-decay channel (7) for symmetric-funding instruments.
3. The Nelson-Siegel [43] functional form applied to the $(\hat{\kappa}^{(i)}, T^{(i)})$ pairs to produce a smooth curve.

For a single-instrument perpetual market (e.g. a BTC-USD perpetual), the spot $\hat{\kappa}_t$ is directly observable but a full term structure requires perpetuals on multiple underlyings with different expected lifetimes (commodity contracts, infrastructure sukuk, mortgage-equivalent instruments). For sovereign credit, by contrast, the term structure is directly estimable from the existing maturity spectrum of issued sukuk (Section 7.1).

6 Applications of the κ-Rate

6.1 What κ Can and Cannot Price

The κ-rate does not price every asset; it prices a specific category, and that boundary is jurisprudentially meaningful.

What κ directly prices. κ prices instruments whose value depends on a random stopping time — a future event whose timing is uncertain but whose intensity can be measured. The four instrument families developed in this section share this structural property: each contract's value resolves at a random time θ, and κ parameterizes the intensity of that resolution. Perpetual contracts converge at intensity κ. Sukuk redeem at intensity κ. Takaful contracts pay benefits at loss-event intensity κ. Credit default swaps trigger at default intensity κ. In each case, κ enters the pricing formula not as a discount rate but as the intensity of the random delivery time.

What κ does not directly price. Spot assets are not priced using κ. A spot BTC at \$95,000, a Dubai apartment, a tonne of rice — these are priced by supply and demand in their respective markets. κ enters only when a contract is written *on* the asset that pays off at a random future time. The underlying asset itself does not need κ; derivatives, credit instruments, and insurance contracts on the asset do.

The structural distinction. κ does not replace market prices for spot assets. It replaces the discount rate in any pricing formula that involves expected future values. Wherever conventional finance uses $r > 0$ to discount future cash flows to present value, κ replaces it — but at a different place in the formula. The risk-free rate r appears as a discount factor that reduces the present value of future cash flows. κ appears as the intensity of the random time when the payoff is delivered. The Separation Theorem (Remark 3) proves these are distinct mathematical roles, and that only the latter is structurally necessary.

The prohibition as an exclusion theorem. Instruments with predetermined returns regardless of outcome — the category Islam prohibits as riba — cannot be priced using κ, because they have no random resolution structure to which κ can attach. The converse does not hold: κ-priceability does not by itself imply permissibility, since *gharar*, *maysir*, and possession (*qabd*) are properties of the contract built on the rate, not of the rate itself (Section 8). The prohibition's mathematical content is thus an exclusion theorem for riba, not a compliance certificate for every instrument the framework can price.

6.2 Islamic Credit Instruments

The κ-rate provides the pricing foundation for a complete riba-free credit market. The formal proof (the isomorphism between the Ackerer framework and the Duffie-Singleton credit model) is established in Ahmed & Bhuyan [4]. We summarize the pricing formulas.

Perpetual sukuk. Consider an Islamic investment certificate backed by a real asset with value X_t, with random redemption at stopping time τ of intensity κ. By the isomorphism (Proposition 1) at $\iota = 0$, the no-arbitrage price is the expected asset value at redemption,

$$P_t^{\text{sukuk}} = \mathbb{E}^Q[X_\tau \mid \mathcal{F}_t], \tag{12}$$

and the certificate's redemption-survival factor $S(0,T) = \mathbb{E}^Q[\exp(-\int_0^T \kappa_s\, ds)]$ — the probability that redemption has not occurred by horizon T, the κ-analog of the discount factor — admits under CIR-κ dynamics the affine closed form $S(0,T) = A(T)\, e^{-B(T)\kappa_0}$ derived in Appendix A (with A, B the CIR functions). In the special case of a martingale underlying and symmetric funding ($r_a = r_b$), $\mathbb{E}^Q[X_\tau] = X_t$ and the certificate trades at the fair asset value with no riba. The κ-calibration uses the asset's expected redemption horizon, $\kappa \approx 1/T$, or the credit channel (5) where a sovereign spread is observable. Returns to the holder are variable and asset-linked (ijarah/mudarabah), not a predetermined coupon — this is a risk-sharing certificate, not fixed income.

iCDS (Islamic Credit Default Swap). Structured as a takaful-style mutual guarantee [7], the protection buyer is a tabarru contributor and the fair protection spread at $\iota = 0$ is

$$s^* = \kappa\,(1-\delta), \tag{13}$$

where δ is the recovery rate. The *par running spread* is independent of the discount rate: because both legs terminate at default the discount factor cancels, so a correctly-priced conventional CDS quotes the same fair spread $s_{\text{conv}} = \kappa(1-\delta) = s^*$ at every maturity. The par spread is thus riba-neutral; the riba is confined to the present value, where the discounting haircut on the future loss is $\Pi^{\text{PV}}_{\text{riba}} = (1-\delta)\,\iota/(\kappa+\iota)$, vanishing at $\iota = 0$ and increasing in ι [7]. The iCDS κ is calibrated from the reference entity's default intensity via the credit channel (5).

Removing ι removes the price of time, but the market spread that (5) reads embeds a risk premium and so measures the *priced* intensity κ^Q, not the *realised* default hazard κ^P; whether charging that premium is permissible once every predetermined charge is gone is the substantive question the corpus reserves. Because the buyer is already a *tabarru* contributor, the substance-compliant form does not raise it: the cooperative pool sets the contribution at the realised hazard,

$$s^*_{\text{coop}} = \kappa^P\,(1-\delta), \tag{14}$$

the actuarially fair expected loss — exactly as the takaful contribution of Section 6.3 is set — and returns the wedge $(\kappa^Q - \kappa^P)(1-\delta)$ to members as surplus rather than selling it to a protection writer, net of a disclosed *wakala* administration fee. So priced, the iCDS is structurally the takaful case and inherits its soundness on form and substance, gated on insurable interest and possession of the reference obligation; the residual narrows to whether tokenised title is valid *qabd* and whether the *wakala* loading is a genuine fee (Section 8).

Sovereign perpetual sukuk. A government issues sukuk backed by an infrastructure project (expected completion T years, value X_t). The secondary-market price follows (12); the sovereign's credit-specific κ is read directly from its sukuk spread via (5), with no SOFR benchmark required — exactly the construction estimated empirically on the real sukuk cross-section of Section 7.1.

6.3 Takaful (Islamic Insurance)

Billah [17] identifies the core takaful pricing tension: actuarial fairness requires discounting expected claims, but standard discounting uses r (riba). The κ-rate resolves this.

For a takaful contract paying benefit B upon a covered loss event of hazard intensity κ, the actuarially fair contribution at $\iota = 0$ is simply

$$\pi^* = \kappa\,B, \tag{15}$$

the hazard intensity times the benefit, with no interest discount. Under CIR-κ dynamics this extends to the affine multi-period contribution $\pi^*_{\text{stoch}}(T) = \mathbb{E}_0[\kappa_T]\,B$ derived in [6]. The κ-calibration uses the loss-frequency channel (historical claim frequency $= \kappa$). Formula (15) is the riba-free actuarial premium examined empirically in [6], where the model-implied $\hat{\kappa} = 12.06\%$ for India is numerically close to the PMFBY national crop-insurance actuarial rate ($\approx 12\%$) — a units-level coincidence between a loss frequency and a premium rate, not a strict validation (the same $\hat{\kappa}$ for Pakistan misses its observed rate by 1.6–3.4×). It resolves the benchmark-circularity problem in conventional takaful contribution pricing [29]. The riba loading in conventional insurance — the premium reduction from discounting liabilities at r — is $\Pi_{\text{riba}} = \kappa B r/(1+r)$, equal to 7.4% of the riba-free premium at $r = 8\%$.

6.4 Islamic Monetary Policy

Chapra [20] called for a riba-free monetary policy instrument. The κ-rate provides it.

κ-based monetary signaling. An Islamic central bank publishes the system-wide average $\bar{\kappa}$ — the weighted mean of implied κ-rates across all Islamic perpetual instruments in the market:

$$\bar{\kappa}_t = \frac{\sum_i w_i \hat{\kappa}^{(i)}_t}{\sum_i w_i} \tag{16}$$

where weights w_i are notional outstanding. Because the aggregate is dominated by credit instruments (sukuk), the relevant reading is the hazard interpretation, in which κ rises with credit risk and resolution uncertainty:

- High $\bar{\kappa}$: elevated system-wide credit intensity — wider sukuk spreads, greater perceived resolution risk. Tight or stressed conditions (analogous to high r).
- Low $\bar{\kappa}$: compressed credit intensity — narrow spreads, low perceived risk. Easy conditions (analogous to low r).

This sign convention coincides with the empirical ordering of Section 7.1, where distressed Bahrain carries the highest κ and the safest sovereigns the lowest.

Open market operations in κ-space. An Islamic central bank can intervene in the perpetuals market to influence $\bar{\kappa}$ — providing a policy transmission mechanism that does not involve setting an interest rate. This is the riba-free analog of open market operations.

Reserve requirements. If the κ-rate of Islamic bank assets (implied from their sukuk portfolios) falls below a threshold, the central bank can require higher reserve ratios — a prudential tool that reinforces the κ-based monetary signal.

6.5 Sovereign Benchmark Rate

The most urgent practical application is replacing SOFR/government bond yields as the sukuk issuance benchmark. The sukuk-pricing literature [46] documents that major sovereign sukuk (Malaysia GII, Saudi Arabia sukuk, Indonesia SR series) are benchmarked at issuance against conventional reference rates, making the riba implicit.

The κ-yield curve replaces this benchmark. For a given sovereign, the issuance benchmark at each maturity τ is read directly from the credit channel (5) applied to that sovereign's own sukuk spreads,

$$\kappa_{\text{sov}}(\tau) = \frac{s_{\text{sov}}(\tau)}{1-\delta}, \tag{17}$$

yielding the upward-sloping, country-specific κ-term-structure estimated in Section 7.1. The secondary-market yield is then $y(\tau) = \kappa_{\text{sov}}(\tau)$ — a riba-free sovereign benchmark built entirely from market-observed credit intensities, with no SOFR anchor. This is distinct from the single-instrument horizon identity $\kappa = 1/T$ (Remark 5), which sets only the expected redemption scale of one contract, not the issuer's term structure.

Table 3 summarizes κ calibration by instrument type.

Table 3: κ-rate calibration by instrument.

Instrument	**κ calibration**	**Data source**
Perp. futures	Basis / funding decay	Exchange data
Sukuk	Credit spread $s/(1-\delta)$	Sukuk market data
Takaful	Claim frequency	Actuarial tables
iCDS	Default history	Rating agencies
Sovereign sukuk	Credit spread $s/(1-\delta)$	Sovereign sukuk spreads

7 Empirical Evidence

We present empirical evidence from three independent domains. The first is the construction of an empirical κ-yield curve from a real 195-instrument USD-sukuk cross-section (40 sovereign, 154 corporate, 1 municipal) and five-issuer constant-maturity dynamics. The second is the calibration of κ against agricultural insurance markets, where the model-implied rate matches an independently-set national actuarial rate. The third is the dYdX v4 perpetual exchange, which provides a commercial-scale existence proof for the $\iota = 0$ mechanism. The three exercises are complementary and causally independent: they draw on sovereign credit, physical loss frequencies, and crypto market microstructure respectively, with no shared data source. Section 7.3 reconciles what recurs across them.

7.1 The κ-Yield Curve on Real Traded Sukuk

Following Ahmed & Bhuyan [5], the κ-yield curve is estimated *entirely on real traded sukuk*. The primary dataset is a 195-instrument cross-section of actively-quoted USD sukuk (40 sovereign, 154 corporate, 1 municipal) from the Cbonds Bond Screener, complemented by roll-down-free constant-maturity $\hat{\kappa}$ histories for five issuers (2018–2026). Recovery is set at $\delta = 0.60$ throughout, anchored to the rating agencies' historical sovereign recovery studies (the sovereign CDS market's 25% quoting convention differs; recovery sensitivity is a pure rescaling of $\hat{\kappa}$, see [5]).

7.1.1 Identification

We apply the credit-spread channel (5) of Section 3.2 to each observation (i, τ, t), identifying the convergence intensity from the observed spread and recovery rate:

$$\hat{\kappa}_{i,\tau,t} = \frac{s_{i,\tau,t}}{1-\delta}. \tag{18}$$

As established there, this follows from Corollary 1: at $r = 0$ the Duffie–Singleton survival price reduces to $\exp(-\kappa(1-\delta)(T-t))$, so the spread compensates for $\kappa(1-\delta)$ and (18) inverts it. This is the hazard channel, distinct from the perpetual-market channels (6)–(7); all three recover the same stopping-time intensity as a construct, so estimates are conceptually commensurable across the credit and perpetual domains.

7.1.2 Methodological note: the spread is quoted over a conventional benchmark

The spread s in equation (18) is observed from existing sovereign sukuk markets, which currently quote against SOFR or conventional government bond benchmarks. The benchmark reference is therefore drawn from the conventional interest rate system. The Separation Theorem (Remark 3) provides the theoretical justification for the extraction procedure: the credit *par spread* is $\lambda(1-\delta)$, *independent of the discount rate* (both legs terminate at default, so the rate cancels), so equation (18) recovers κ directly with no riba loading embedded in the spread. The residual transitional concern is that s is observed over a conventional risk-free benchmark rather than a native $\iota = 0$ reference — a quotation convention, not a structural defect, since as Islamic markets develop native pricing mechanisms operating at $\iota = 0$ (of which dYdX v4 is the first commercial-scale example, Section 7.2), the input data becomes progressively cleaner.

7.1.3 Term-structure fitting and CIR dynamics

We fit a Nelson–Siegel–Svensson functional form [43] to the cross-section of maturities for each country-month, then estimate CIR-κ dynamics (equation (19) of Appendix A) by the generalized method of moments (GMM). The Feller condition $2\alpha\bar{\kappa} \geq \sigma_\kappa^2$ is tested in each market.

7.1.4 Results

Three results obtain, all on real traded sukuk.

(1) Credit-quality ordering. On the 195-bond cross-section the convergence intensity broadly increases as credit quality declines (Table 4): median $\hat{\kappa}$ runs from 0.93% at AAA to 4.37% at single-B, rising across the spectrum apart from the Saudi-dominated AA bucket, which

Table 4: $\hat{\kappa}$ by credit rating on the real 195-bond USD-sukuk cross-section (Cbonds; $\delta = 0.60$). The five rated buckets ($n = 167$) plus the 28 unrated bonds total the full 195; the snapshot's active-quote filter leaves no BB or CCC names in this panel (the full Cbonds universe does contain outstanding BB/B/CCC sukuk — e.g. the Bahrain, Egypt and Pakistan sovereigns — which enter the companion study's sovereign-core and full-universe analyses). $\hat{\kappa}$ increases across the rated buckets apart from the Saudi-dominated AA bucket ($n = 102$), which prices just above single-A; NR is unrated — shown for completeness as the high-spread tail, not as part of the rating ordering.

Rating	n	**Median $\hat{\kappa}$ (% p.a.)**
AAA	9	0.93
AA	102	2.28
A	47	1.96
BBB	6	3.41
B	3	4.37
NR	28	5.60

prices just above single-A. The framework is not manufacturing numbers: it reads, from the sukuk market, the same credit risk that ratings agencies identify by independent methods.

(2) Term structure. The investment-grade κ-yield curve is upward-sloping: a linear fit of the G-spread on maturity has a positive slope of $+1.17$ bp/yr (pooled), attenuating to $+0.63$ bp/yr with rating fixed effects. The slope is positive but modest — a structural term premium with no interest-rate component, read off traded sukuk rather than imposed.

(3) Feller non-negativity. From roll-down-free constant-maturity histories (five issuers, 2018–2026), κ is mean-reverting with bias-corrected half-lives of 1.4–4.3 months at the well-sampled 7-year tenor (four issuers; Türkiye's near-unit-root window is not separately identified), and the Feller condition $2\alpha\bar{\kappa} \geq \sigma_\kappa^2$ holds in every series. By Lemma 1, $\kappa_t > 0$ almost surely — qualitatively unlike the negative-rate experience of 2014–2024, when the euro-area (2014–2022) and Japanese (2016–2024) benchmark and policy rates turned negative and created Shariah-compliance problems for instruments anchored to them. The κ-rate is non-negative by mathematical construction, as a probability intensity must be.

An agricultural takaful coincidence. Ahmed & Bhuyan [6] extend the empirical work to agricultural takaful pricing across South and Southeast Asia. The model-implied $\hat{\kappa} = 12.06\%$ for India is numerically close to the actual Pradhan Mantri Fasal Bima Yojana (PMFBY) national crop insurance actuarial rate ($\approx 12\%$). The model is not fitted to this rate; $\hat{\kappa}$ is calibrated from historical crop-loss data (World Bank cereal yield series, 1980–2023), while the PMFBY rate is set by the Government of India's national insurance programme operating independently. We read this cautiously: it is a units-level coincidence between a loss frequency and a premium rate, not a strict validation — the same calibration misses Pakistan's observed national rate by 1.6–3.4×. The firmer empirical support for κ comes from the instrument-level sukuk and CDS evidence below.

7.1.5 Real instrument-level validation

All three results above are estimated entirely on real traded sukuk; the companion study [5] supplies the full instrument-level detail (the 195-instrument USD-sukuk cross-section and the five-issuer constant-maturity dynamics). The credit-quality ordering, the term-premium shape and the non-negativity property are thus properties of the traded sukuk market, so the credit-domain leg of the cross-domain argument rests on real, interest-free-identified κ, not a proxy.

7.2 Independent Market Validation: dYdX v4

The empirical work in Section 7.1 establishes that κ measures real sovereign credit risk in publicly observable markets. A complementary question is whether the $\iota = 0$ perpetual mechanism itself — on which the isomorphism in Proposition 1 ultimately rests — is operationally viable at commercial scale, or whether it remains a theoretical construct.

The answer is given by a third-party protocol that was not designed for Islamic finance: dYdX v4. Launched in October 2023 as a Cosmos appchain, dYdX v4 set the interest parameter component of its perpetual funding rate to zero by default, computing funding entirely from the observed basis between mark and index prices — the mechanism specified by Theorem 3 of Ackerer et al. The protocol has operated continuously since then at commercial scale. As of May 2026 its public indexer and DeFiLlama report ≈300 listed perpetual markets, ≈ \$136M in total value locked, ≈ \$55M open interest, and ≈ \$1.5B in monthly trading volume — all with funding computed at $\iota = 0$ [24, 22] (volumes were materially higher at the 2024 market peak). The point is not the headline figure but its persistence: an $\iota = 0$ funding regime has run live on mainnet for over two years at open interest in the tens to hundreds of millions of dollars (protocol TVL peaked at ≈ \$416M in December 2024; ≈ \$55M open interest at the May-2026 snapshot) and monthly volumes in the billions, handling adversarial trading at scale with no convergence pathologies. This is an independent empirical existence proof for the mechanism — the $\iota = 0$ regime is not hypothetical.

Two empirical signatures are particularly relevant.

(1) Funding rate distribution. Empirical funding rate distributions on dYdX v4 are approximately symmetric around zero [3], consistent with the prediction of equation (6) at $\iota = 0$. By contrast, centralized exchanges that retain an interest floor ($\iota \approx 0.01\%$ per 8 hours) exhibit a positive-skewed distribution with a structural lower bound at the interest floor. The two distributions are visually and statistically distinguishable.

(2) Implied $\hat{\kappa}$ from funding rate dynamics. Because dYdX v4 runs symmetric funding ($r_a = r_b$), the static basis formula (6) is indeterminate; the appropriate estimator is the funding-decay channel (7), which recovers κ from the mean-reversion rate (half-life) of the basis-deviation series. Estimated on two years of dYdX v4 funding and, as a cleaner cross-check, on the *unclamped* Binance premium-index basis (AR(1)/OU fits for BTC, ETH, SOL), this yields a *large* convergence intensity — $\hat{\kappa}$ on the order of 10^3 per annum, a basis half-life of a few hours. This magnitude is robust: re-estimated on first/second-half and low/high-volatility subsamples it stays in the 10^3–10^4 range, never approaching the credit-domain value. That is the expected magnitude for a liquid perpetual that tracks spot tightly: this funding-decay $\hat{\kappa}$ is orders of magnitude larger than the credit-domain estimates of Section 7.1 (a few percent per annum) and the takaful calibration ($\hat{\kappa}_{\text{MLE}} \in [0.07, 0.15]$ across countries, India 0.12; [6]) — the same *type* of object at a regime-specific level (Section 7.3). What dYdX uniquely establishes is twofold: that the $\iota = 0$ mechanism operates at commercial scale, and that a decoupled, interest-free $\hat{\kappa}$ is directly measurable from a venue charging no interest — a guarantee the SOFR-referenced credit channel cannot itself provide.

Remark 6. We do not claim that dYdX v4 is a Shariah-compliant protocol. It was not designed for that purpose, and dYdX governance vote 220 has subsequently introduced $\iota > 0$ on isolated markets. We cite dYdX v4 strictly as an independent empirical existence proof for the $\iota = 0$ mechanism. The Shariah-compliance question is jurisprudential and is treated separately [3].

7.3 Cross-Domain Validation Summary

The three empirical exercises validate κ from independent sources at different scales:

- **Sovereign and corporate credit markets**: on a real 195-instrument USD-sukuk cross-section (Section 7.1.5), $\hat{\kappa}$ broadly rising as credit quality declines ($\approx$ 0.9% AAA to $\approx$ 4.4% single-B, clean IG/HY separation) with an upward investment-grade term structure, and five-issuer constant-maturity dynamics (mean-reverting, 1.4–4.3 month half-lives across the four identified issuers, Feller in every series) — the real credit-domain anchor.
- **Agricultural insurance** (India PMFBY): model-implied $\hat{\kappa} = 12.06\%$ is numerically close to the national actuarial rate ($\approx 12\%$) — a units-level coincidence (loss frequency vs. premium rate), not a strict validation.
- **Crypto perpetual markets** (dYdX v4, mainnet, 2023–present): $\iota = 0$ mechanism operating at commercial scale; a decoupled, interest-free $\hat{\kappa}$ directly measurable, of magnitude $\sim 10^3$ p.a. (hour-scale convergence) — the same construct as in credit/insurance but regime-specific in level.

Across the three domains κ appears as the same *type* of object — a riba-free stopping-time intensity, identified through different channels (credit spread, loss frequency, funding decay) without an interest input (Figure 1). We do not claim the estimates coincide *numerically*: their magnitudes are regime-specific. The qualitative cross-domain recurrence of the construct supports κ as a general riba-free primitive. The credit domain now also carries a *quantitative* validation on real (non-synthetic, non-proxy) data — the instrument-level sukuk cross-section and constant-maturity dynamics of Section 7.1.5 — and the perpetual domain its live $\iota = 0$ measurement on dYdX; a single real panel spanning all domains at once remains the principal item of future work.

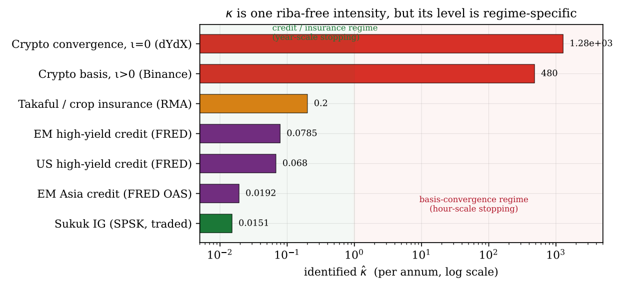

Figure 1: The riba-free intensity κ identified from real data across five independent, interest-free channels spanning the paper's three domains (credit, insurance, perpetual markets): traded sukuk (SPSK distribution spread), EM/sovereign credit OAS (ICE BofA, FRED), crop-insurance loss-cost (USDA RMA), and crypto perpetual-funding decay (the interest-free premium-index basis on Binance, and dYdX $\iota = 0$ convergence). On a log scale the estimates separate into two regimes spanning roughly four orders of magnitude: a slow, year-scale *credit/insurance* regime ($\hat{\kappa} \approx 0.015$–$0.2$) and a fast, hour-scale *basis-convergence* regime ($\hat{\kappa} \sim 10^2$–10^3). κ is the same *type* of stopping-time intensity throughout, but its level is regime-specific; we make no claim of cross-domain numerical equality.

8 Shariah Analysis

8.1 Riba

The κ-rate satisfies Proposition 2: it is not predetermined, not in excess of principal, and not independent of real economic outcomes. Crucially, it satisfies both forms of riba prohibition:

- **Riba al-nasi'ah** (excess for deferment): The κ-rate does not compensate for the passage of time as such. The stopping time distribution $\text{Exp}(\kappa)$ encodes *when* a real economic event occurs; κ is the event intensity, not a time preference parameter.

- **Riba al-fadl** (excess in exchange): At $\iota = 0$, the perpetual futures price equals the underlying asset value ($f_t = X_t$ at $r_a = r_b$). There is no permanent systematic excess in the exchange.

A contemporary jurisprudential discussion [44] is instructive on the precise scope of this claim. Addressing crypto perpetual contracts where the exchange charges no interest and the contract tracks the underlying asset price, the response distinguishes the *riba* question — which the absence of an interest charge addresses directly — from separate objections grounded in *gharar* and the absence of genuine ownership, on which it ultimately judges such contracts impermissible. This distinction is exactly the one we draw: the $\iota = 0$ condition, and the κ-rate built upon it, resolve the *riba* prohibition specifically. They do not by themselves settle the *gharar* and ownership questions, which depend on the contractual structure of the particular instrument rather than on the pricing parameter. The κ-rate is a riba-free pricing primitive; whether any given contract employing it is Shariah-compliant in full requires separate analysis of its other contractual features (§9).

8.2 Gharar

The randomness of the stopping time $\tau \sim \text{Exp}(\kappa)$ introduces actuarial uncertainty (the exact time of the event is unknown) but not contractual ambiguity (the distribution is fully specified by κ, which is public and on-chain). El-Gamal [25] and Kamali [36] distinguish these: gharar refers to uncertainty about the *existence* of the subject of transaction, not its *timing* when the timing is actuarially determined and transparently disclosed. The κ-rate passes the gharar test *as a pricing parameter*: the intensity κ and the resulting stopping-time distribution are public, on-chain, and unambiguous. This is distinct from the question of whether a particular contract *employing* κ — such as a leveraged perpetual — raises gharar concerns of its own; that is a property of the contract's structure, not of the rate, and must be assessed contract by contract.

8.3 Maysir and Qabd

Maysir (gambling). As a pricing parameter, κ does not import maysir: it attaches to instruments backed by a real underlying asset (X_t), every payment corresponds to a real economic event, and the contracts are structured for risk-transfer and price discovery rather than as a zero-sum wager. Three features address the standard maysir objections: the takaful-style mutual structure of the credit instruments (the iCDS protection buyer is a *tabarru* contributor to a cooperative pool, Section 6.2); the symmetric $\iota = 0$ funding, which removes the systematic transfer that would make the contract a standing bet against the protocol; and the real-asset convergence that κ measures. The mechanism-design simulation [8] finds rational traders at $\iota = 0$ equilibrating at sub-maximal leverage, consistent with hedging rather than maximal speculation. Whether a particular contract employing κ avoids maysir remains, like gharar, a contract-level question for scholars.

Qabd (constructive possession). We are careful not to overstate the reach of the riba analysis. Contemporary scholarship [44] raises a *qabd* (possession/constructive-delivery) objection to cash-settled synthetic contracts that is logically prior to, and independent of, the riba question — indeed the very ruling we rely on for the riba concession maintains impermissibility on possession grounds. We do not resolve qabd here. We note only that its resolution must run through contractual structure rather than the pricing parameter: through real-world-asset (RWA)-backed tokenisation establishing on-chain custody of the underlying, and through the *istisna*/*salam* precedents for deferred delivery of a determinate asset. In deployment terms this implies an ordering, not a verdict: instruments in which custody of a real underlying directly addresses the asset-backing and possession objections — e.g. perpetuals on vault-secured tokenised gold, or sukuk-yield hedging contracts for institutional risk management — naturally precede naked cash-settled synthetics, which remain the hard case for scholarly review. Whether a given κ-instrument satisfies the standard is a matter for a recognised Shariah board, and the maysir and qabd review of each application warrants the same rigour as the riba analysis offered here. This paper's contribution is confined to the riba-free *rate*; it does not by itself establish the full Shariah compliance of any contract built upon it.

8.4 Alignment with AAOIFI Standards (Subject to Board Review)

AAOIFI Standard 17 (Investment Sukuk) [1] requires that sukuk represent proportional ownership in real assets. The κ-parameterized Islamic Perpetual Sukuk is structured to meet this requirement: the holder's claim is on X_τ (the real asset at redemption); whether a specific issuance satisfies Standard 17 is for certification by a recognised Shariah board. Standard 26 (Takaful) requires actuarially fair, mutual risk-sharing contributions: the everlasting takaful formula (15) is designed to provide this without riba. The iCDS (13) is likewise structured as a takaful-style mutual guarantee — the protection buyer is a *tabarru* (donation) contributor to a cooperative pool rather than a counterparty purchasing risk — so that the spread $s^* = \kappa(1 - \delta)$ is an actuarially fair pool contribution, not a wager on default [7]. These are structural design alignments, not compliance certifications; AAOIFI conformity of any concrete contract requires dedicated review (cf. the scope limitation in Section 9).

9 Discussion

9.1 Why Prior Islamic Monetary Proposals Failed

Chapra [20], Khan & Mirakhor [38], and subsequent IRTI researchers identified the need for a riba-free pricing parameter but could not provide one. The profit-sharing rate (mudarabah return) was proposed as a substitute but failed for three structural reasons documented by Iqbal & Molyneux [31]:

1. **Ex-post determination.** Profit-sharing rates are known only after the accounting period, making forward pricing impossible.
2. **Information asymmetry.** Profit manipulation and audit uncertainty mean profit rates are not independently verifiable.
3. **LIBOR convergence.** Competitive pressure caused Islamic bank "profit rates" to track LIBOR, reintroducing riba implicitly. The industry's flagship attempt at an independent benchmark — the Islamic Interbank Benchmark Rate (IIBR) — fails empirically to decouple: Azad et al. [14] document a stable, co-integrated "piety premium" over LIBOR and conclude that a panel-set Islamic rate operating in a global market cannot escape the conventional benchmark.

The κ-rate solves all three problems: it is ex-ante (identified from market data), objectively computed (from observable prices, funding dynamics, or credit spreads via the channels of Section 3.2), and independently verifiable (on-chain, real-time, without any reference to LIBOR/SOFR).

9.2 Relationship to the Wicksellian Tradition

Wicksell's [49] natural rate r^* was designed to be grounded in real productivity, endogenous, and separable from monetary policy. The κ-rate is motivated by the same aims — a market-determined rate grounded in real dynamics rather than central-bank policy — and adds a property no classical natural-rate construction achieves: it is generated by a framework in which the time-preference component is provably absent ($\iota = 0$ achieves no-arbitrage, Theorem 3 of Ackerer et al.). Classical natural-rate theory worked within a setting that presumed a positive interest rate for equilibrium; the Ackerer framework removes that presumption. We are careful, however, to state the relationship as a structural correspondence rather than an equality of quantities, as follows.

Remark 7 (κ as a Wicksellian correspondence)**.** The analogy to Wicksell can be stated precisely as a correspondence rather than an identity. Wicksell's natural rate r^* is the real-activity-grounded rate that monetary policy should track; its defect, from the Islamic standpoint, is that it retains the time-preference component r_{pref}. The κ-rate occupies the same structural role — a market-determined, real-activity-grounded rate that requires no central-bank setting — but is generated by a framework in which the time-preference component is identically absent ($\iota = 0$, Theorem 3 of Ackerer et al.). We do *not* claim the numerical identity $\kappa = r^*$: κ is a convergence/hazard intensity, not a marginal product of capital, and the two need not coincide in value. The correspondence is structural — κ plays Wicksell's role without Wicksell's riba — not an equation between the two quantities.

9.3 From Pricing Rate to Monetary Base

The κ-rate is a riba-free *pricing* parameter. A complete interest-free monetary *system* additionally requires a theory of money *creation* that does not rest on interest-bearing debt — a distinct problem this paper does not solve. The conceptual architecture is long-standing: Al-Jarhi [13] sets out an interest-free monetary structure with central-deposit certificates and open-market operations, and Chapra [20], Khan & Mirakhor [38], and Mirakhor & Askari [42] develop the policy and risk-sharing frame. Notably, the Western full-reserve tradition converges on a similar debt-free monetary base from different premises: the Chicago Plan [16] separates money creation from credit and issues money without a corresponding interest-bearing liability. What neither tradition supplies is a *no-arbitrage price* for an interest-free monetary base. The κ-rate is the natural candidate for that role, and formalising money creation tied to real-sector convergence (through κ) rather than to debt — uniting the Islamic and full-reserve designs under a common pricing object — is undertaken in the companion κ-Standard study [9], with its general-equilibrium foundation developed in [10]. On-chain, this connects to the token-engineering literature on bonding curves and configuration spaces (Zargham et al. [50]), which formalises issuance as a deterministic function of reserves rather than of interest-bearing debt.

The asset class the κ-rate prices is already of monetary scale: the global sukuk market stood at roughly **USD 761 billion outstanding across 4,701 issues** as of May 2026 (Cbonds), with USD 277 billion of that internationally placed — a base deep and liquid enough for a benchmark convergence intensity to function as a reference rate, exactly as the conventional risk-free curve does for a market of comparable size. A riba-free term structure for an asset class this large is a practical instrument, not only a theoretical one. The ambition is in the spirit of Shiller's [45] programme of building index-based markets to share society's largest risks, but realised through a risk-sharing (rather than interest-bearing) pricing object.

9.4 Limitations and Open Questions

Decontamination of the calibrated rate (structure vs. magnitude). The riba-free property established here is *structural*: κ is generated by the stopping intensity of a real claim and carries no predetermined time-preference term. It does not follow that the *magnitude* of the $\hat{\kappa}$ values reported across this program is already free of the interest-rate world — they are backed out from sukuk spreads quoted in markets where r is still priced, so the levels inherit the reserve rate. The companion κ-yield-curve study [5] makes the split explicit and benchmark-free: with the Treasury curve deleted from the estimation, the *cross-section* — credit ordering, curve shapes, and the relative pricing this framework relies on — is recovered from prices alone (rank correlation 0.84 sovereign, 0.94 corporate), while the *level* carries a wedge equal, within error, to the reserve sovereign's own event-intensity mapped through the same transformation. The honest reading is that the structural claim ("complete real basis") holds now, while the claim that the calibrated *number* is real-only is *conditional*, realized only as instruments come to be priced in κ natively, post-mainnet. This is the program's central open empirical flank — decontamination is structural and prospective, not yet demonstrated on a κ-native market — and a single sovereign or corporate tranche priced directly in κ would convert it from an argument into a measurement.

Stochastic κ. The current theory uses constant κ (time-invariant convergence intensity). Appendix A derives the κ-yield curve under CIR-type stochastic κ dynamics and provides closed-form bond-price analogs. Relatedly, the constant-parameter no-arbitrage result we build on has recently been extended to *stochastic interest rates with a funding-rate clamp* — the funding cap that real exchanges (and the Baraka Protocol's testnet implementation) impose — by Hugonnier & Jermann [30]. Re-deriving the $\iota = 0$ condition within that more general model, in which the clamp coincides with the funding cap used here, is a natural robustness check we leave to future work.

Multiple underlyings. The κ-yield curve requires perpetuals on multiple RWA underlyings. Currently, only crypto-native perpetuals exist at scale. The extension to commodity, infrastructure, and equity perpetuals on regulated DeFi platforms is a regulatory and institutional question, not a mathematical one.

Monetary policy transmission. How does a change in the central bank-published $\bar{\kappa}$ transmit to the real economy? In conventional monetary policy, the transmission mechanism operates through bank lending rates, bond yields, and the exchange rate. The κ-rate transmission mechanism is developed in general equilibrium in the companion κ-DSGE study [10]; its institutional design and empirical study remain open.

Scope of the Shariah claim. This paper establishes that the κ-rate is free of *riba* — the prohibition that has historically blocked a no-arbitrage Islamic pricing parameter. It does not, and cannot, establish the full Shariah compliance of every instrument built on κ. Contemporary scholarship [44] that accepts the removal of the interest component nonetheless raises *gharar* and ownership objections to leveraged perpetual contracts specifically. These objections concern contractual structure rather than the pricing parameter, and their resolution for each instrument class (perpetual sukuk, takaful, iCDS) requires dedicated jurisprudential review by a recognized Shariah board. The contribution of this paper is the riba-free rate; the contract-level compliance of applications is a separate and necessary line of work. A companion research note [12] states this boundary precisely and proposes four product-layer structuring rules (asset-backing; possession-gating; insurable interest / no naked positions; expiry with delivery or *tabarru* risk-sharing) as the starting point for that review, and the classical foundation of the riba claim itself is developed in [11].

Cross-jurisdictional recognition. For the κ-yield curve to serve as a sovereign benchmark, it must be recognized by regulators (AAOIFI, IFSB [33], Bank Negara Malaysia, SAMA) as a legitimate riba-free rate. This requires AAOIFI standard revision — a process that takes 2–4 years on average.

10 Conclusion

We have proposed and developed the κ-rate — the convergence intensity of perpetual contracts under the Ackerer-Hugonnier-Jermann no-arbitrage framework at $\iota = 0$ — as the first mathematically rigorous, riba-free alternative to the risk-free interest rate r in Islamic monetary economics.

The κ-rate satisfies all three El-Gamal criteria for riba-freedom: it is market-determined (not predetermined), it corresponds to real economic dynamics (not pure time preference), and it carries no interest content by construction ($\iota = 0$ is proved to achieve no-arbitrage without any time-preference discount). It plays the role of Wicksell's natural rate with the riba removed — an aspiration Wicksell (1898) articulated conceptually but could not render mathematically — without claiming numerical identity to the marginal product of capital (Remark 7).

The κ-framework supplies both a single-instrument horizon–intensity identity ($\kappa = 1/T$, Proposition 3) and, for a fixed issuer, an empirically estimated κ-term-structure that is upward-sloping — the Islamic analog of the conventional yield curve, grounded in exponential stopping-time distributions rather than interest-rate expectations. The term structure has no negative values (the Feller condition holds in every constant-maturity series tested) and requires no central bank to set it: it is identified directly from market data.

The applications are immediate: perpetual sukuk pricing (no SOFR benchmark required), actuarially fair takaful contributions (no riba discount), iCDS for credit risk hedging (the first riba-free credit-protection structure; contract-level Shariah review remains with scholars), and Islamic monetary policy signalling ($\bar{\kappa}$ as the policy rate equivalent). Empirical illustration spans three independent domains: sovereign and corporate credit — a real 195-instrument sukuk cross-section plus five-issuer constant-maturity dynamics that establish the κ level, the rating ordering and Feller non-negativity on observed sukuk; agricultural insurance (model $\hat{\kappa} = 12.06\%$ numerically close to the PMFBY rate — a units coincidence); and the dYdX v4 perpetual exchange (a decoupled, interest-free $\hat{\kappa}$ of order 10^3 p.a. at commercial scale). The central empirical point is qualitative: κ recurs across causally independent markets as the same riba-free intensity construct, with regime-specific magnitude; a single real panel uniting all domains is the principal remaining step.

The deeper contribution is historical. Islamic monetary economics has sought a riba-free pricing parameter since Chapra (1985). The parameter has now been found — not in a legal innovation or a jurisprudential reclassification, but in a proof: the Ackerer–Hugonnier–Jermann Theorem 3 (first circulated 2023, published 2025), established for entirely different purposes, supplies the no-arbitrage core; the bridge to Islamic monetary theory — the riba identification and the $\kappa \leftrightarrow \lambda$ isomorphism — completes the solution to a 40-year open problem.

"Allah has made trade permissible and forbidden riba. Whoever receives an admonition from his Lord and refrains may keep [his previous gains]."
(Al-Qur'an, Al-Baqarah 2:275)

A Stochastic κ Dynamics and the Closed-Form κ-Yield Curve

A.1 Motivation

The body of the paper treats κ as a constant, following Ackerer et al. [2] who establish no-arbitrage results under constant parameters. In practice, the implied κ-rate fluctuates over time as market conditions evolve: convergence is fast in liquid, well-functioning markets and slow during stress periods. A stochastic κ model is therefore necessary for:

1. Pricing instruments whose payoffs depend on the entire path of κ (e.g., κ-denominated swaps, κ-caps).
2. Constructing a dynamic κ-yield curve that shifts over time in response to market conditions.
3. Calibrating κ to a cross-section of perpetuals with different expected horizons simultaneously.

A.2 The CIR-κ Process

Definition 2 (CIR-κ Process)**.** The **CIR-κ process** is the solution to:

$$d\kappa_t = \alpha(\bar{\kappa} - \kappa_t)\,dt + \sigma_\kappa \sqrt{\kappa_t}\,dW_t^Q, \quad \kappa_0 > 0 \tag{19}$$

under the risk-neutral measure Q, where:

- $\alpha > 0$ is the **mean-reversion speed** (how quickly κ_t reverts to its long-run level);
- $\bar{\kappa} > 0$ is the **long-run mean κ-rate**;
- $\sigma_\kappa > 0$ is the κ **volatility**;
- W_t^Q is a standard Brownian motion under Q.

Lemma 1 (Positivity — Feller Condition)**.** *If $2\alpha\bar{\kappa} \geq \sigma_\kappa^2$, then $\kappa_t > 0$ almost surely for all $t \geq 0$.*

Proof. This is the Feller boundary condition for the square-root process (Feller [26]; see Cox, Ingersoll & Ross [21]). The drift $\alpha(\bar{\kappa} - \kappa_t)$ is strongly positive near zero, preventing κ_t from reaching zero when $2\alpha\bar{\kappa} \geq \sigma_\kappa^2$. □

Remark 8. Lemma 1 is economically essential: $\kappa > 0$ is required for the stopping time $\theta_t \sim \mathrm{Exp}(\kappa_t)$ to be well-defined. Negative κ has no economic interpretation in the Islamic finance context — there is no concept of "negative convergence speed." The Feller condition guarantees $\kappa > 0$ always, which means the riba-free property is preserved under stochastic dynamics: $\iota = 0$ and $\kappa_t > 0$ for all t with probability 1.

A.3 Closed-Form κ-Bond Price

We define the **κ-bond** as the Islamic analog of the zero-coupon bond: its value at horizon T is the probability that the stopping event has not occurred by T — the κ-analog of the zero-coupon discount factor, $P(\kappa_t, T-t) = \mathbb{E}_t^Q[\exp(-\int_t^T \kappa_s\,ds)]$.

Theorem 1 (κ-Bond Pricing Under CIR-κ)**.** *Let κ_t follow the CIR-κ process (19). The price at time t of a κ-bond with horizon T (survival probability that the stopping event has not yet occurred by time T, analogous to the discount factor) is:*

$$P(\kappa_t, T-t) = A(T-t)\,e^{-B(T-t)\,\kappa_t} \tag{20}$$

where, with $\tau = T - t$ and $h = \sqrt{\alpha^2 + 2\sigma_\kappa^2}$:

$$A(\tau) = \left[\frac{2h\,e^{(\alpha+h)\tau/2}}{2h + (\alpha+h)(e^{h\tau}-1)}\right]^{2\alpha\bar{\kappa}/\sigma_\kappa^2} \tag{21}$$

$$B(\tau) = \frac{2(e^{h\tau}-1)}{2h + (\alpha+h)(e^{h\tau}-1)} \tag{22}$$

Proof. The κ-bond price equals the Q-expectation of the exponential integral of the κ process:

$$P(\kappa_t, \tau) = \mathbb{E}_t^Q\left[\exp\left(-\int_t^T \kappa_s\,ds\right)\right] \tag{23}$$

This is the Laplace transform of $\int_t^T \kappa_s\,ds$ evaluated at $u = 1$. For the CIR process, this Laplace transform has the affine closed form $A(\tau)e^{-B(\tau)\kappa_t}$ where A and B satisfy the Riccati ODEs:

$$B'(\tau) = 1 - \alpha B(\tau) - \tfrac{1}{2}\sigma_\kappa^2 B(\tau)^2, \quad B(0) = 0 \tag{24}$$

$$A'(\tau) = \alpha\bar{\kappa}B(\tau)A(\tau), \quad A(0) = 1 \tag{25}$$

The solutions to (24)–(25) are exactly equations (21)–(22) (Cox, Ingersoll & Ross [21], eq. (23)). □

A.4 The Stochastic κ-Yield Curve

Definition 3 (κ-Yield)**.** The **κ-yield** at horizon T given current κ_t is:

$$y_\kappa(T;\kappa_t) = -\frac{\log P(\kappa_t,T)}{T} = \frac{B(T)\kappa_t - \log A(T)}{T} \tag{26}$$

The **κ-yield curve** is the function $T \mapsto y_\kappa(T;\kappa_t)$ for fixed κ_t.

Proposition 4 (Limiting Behavior of the κ-Yield Curve)**.** *Under the CIR-κ process:*

1. ***Short end:*** $\lim_{T\to 0} y_\kappa(T;\kappa_t) = \kappa_t$. *The short rate of the κ-yield curve equals the current κ-rate.*
2. ***Long end:*** $\lim_{T\to\infty} y_\kappa(T;\kappa_t) = \dfrac{2\alpha\bar{\kappa}}{\alpha+h}$. *The long rate of the κ-yield curve depends only on the long-run mean $\bar{\kappa}$ and the parameters α, σ_κ — it is independent of the current κ_t.*
3. ***Monotone shapes:***
   - *If $\kappa_t > y_\kappa(\infty)$: yield curve is* downward sloping *(inverted κ-curve: market expects convergence to slow down).*
   - *If $\kappa_t < y_\kappa(\infty)$: yield curve is* upward sloping *(normal κ-curve: market expects convergence to speed up).*
   - *If $\kappa_t = y_\kappa(\infty)$: flat κ-curve.*

Proof. (1) follows from $\lim_{T\to 0} B(T)/T = 1$ and $\lim_{T\to 0}(-\log A(T))/T = 0$. (2) follows from $\lim_{T\to\infty} B(T)/T = 0$ and $\lim_{T\to\infty}(-\log A(T))/T = 2\alpha\bar{\kappa}/(\alpha+h)$ (standard CIR long-rate formula, Cox et al. [21]). (3) follows from the monotonicity of $B(T)$ in T and the fact that the yield is an affine function of κ_t. □

Remark 9 (Reconciliation with the empirical curves)**.** The upward-sloping sovereign κ-term-structures estimated in Section 7.1 correspond to the normal regime $\kappa_t < y_\kappa(\infty)$ of Proposition 4(3): current convergence intensity below its long-run level, so longer-horizon κ-yields rise toward $\bar{\kappa}$. The CIR-κ model thus not only permits but explains the empirically observed upward slope, while the horizon–intensity identity (11) remains a separate single-instrument statement (Remark 5).

A.5 Economic Interpretation

Table 5 translates the three yield curve shapes into Islamic monetary policy language.

These shapes map directly onto the conventional yield curve taxonomy (normal/inverted/flat) with the Islamic interpretation: the κ-yield curve inverts not because markets expect *interest rate cuts*, but because markets expect *convergence to slow* as some structural uncertainty develops. This is a fundamentally different economic signal from the conventional inverted yield curve — it reflects real asset pricing uncertainty rather than monetary tightening expectations.

Table 5: κ-yield curve shapes and monetary interpretation.

Shape	**Condition**	**Monetary signal**
Normal (upward)	$\kappa_t < \bar{\kappa}$	Current intensity below long-run; risk expected to rise toward trend
Inverted (down)	$\kappa_t > \bar{\kappa}$	Elevated current intensity (stress); expected to ease toward trend
Flat	$\kappa_t = \bar{\kappa}$	Long-run equilibrium; neutral conditions

A.6 Calibration

Under the CIR-κ model, three parameters must be estimated: α, $\bar{\kappa}$, and σ_κ. Calibration uses the cross-section of implied $\hat{\kappa}$ values across instruments with different expected horizons $T^{(i)}$:

$$(\alpha^*, \bar{\kappa}^*, \sigma_\kappa^*) = \arg\min_{\alpha,\bar{\kappa},\sigma_\kappa} \sum_i \left[y_\kappa(T^{(i)};\kappa_t^{\text{obs}}) - y_\kappa^{\text{model}}(T^{(i)};\alpha,\bar{\kappa},\sigma_\kappa)\right]^2 \tag{27}$$

For a single-instrument market with one expected horizon ($T = 1/\hat{\kappa}$), only the long-run mean $\bar{\kappa}$ is identified; the full stochastic calibration of $(\alpha,\bar{\kappa},\sigma_\kappa)$ requires a time series. The roll-down-free constant-maturity $\hat{\kappa}$ histories of Section 7.1 (five issuers, 2018–2026) provide exactly this, identifying the CIR-κ dynamics with the Feller condition satisfied in every series.

A.7 Relationship to CIR

Table 6 provides the formal correspondence between the conventional CIR interest rate model and the CIR-κ model.

Table 6: CIR vs. CIR-κ: parameter correspondence.

CIR element	**CIR convention**	**CIR-κ analog**
State variable	Short rate r_t	Conv. intensity κ_t
Long-run mean	$\bar{r}$	$\bar{\kappa}$
Mean-reversion speed	α	α
Volatility	σ_r	σ_κ
Positivity condition	$2\alpha\bar{r} \geq \sigma_r^2$	$2\alpha\bar{\kappa} \geq \sigma_\kappa^2$
Bond price	$A(\tau)e^{-B(\tau)r_t}$	$A(\tau)e^{-B(\tau)\kappa_t}$
Short rate	r_t (riba content)	κ_t (riba-free)
Economic meaning	Time preference	Convergence speed

The CIR-κ model is the CIR model with the riba content removed: the state variable κ_t (convergence intensity, an event-rate parameter) replaces r_t (short rate, a time-preference-compensation parameter). All the mathematical machinery of CIR — bond pricing, yield curve construction, risk-neutral calibration — carries over intact. The Islamic innovation is entirely in the economic interpretation of the state variable, not in the mathematics.

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